Verifiable C

Applying the Verified Software Toolchain to C programs

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1 Overview

Verifiable C is a language and program logic for reasoning about the functional correctness of C programs. The language is a subset of CompCert C light; it is a dialect of C in which side-effects and loads have been factored out of expressions. The program logic is a higher-order separation logic, a kind of Hoare logic with better support for reasoning about pointer data structures, function pointers, and data abstraction.

Verifiable C is foundationally sound. That is, it is proved (with a machine-checked proof in the Coq proof assistant) that,

Whatever observable property about a C program you prove using the Verifiable C program logic, that property will actually hold on the assembly-language program that comes out of the C compiler.

This soundness proof comes in two parts: The program logic is proved sound with respect to the semantics of CompCert C, by a team of researchers primarily at Princeton University; and the C compiler is proved correct with respect to those same semantics, by a team of researchers primarily at INRIA. This chain of proofs from top to bottom, connected in Coq at specification interfaces, is part of the Verified Software Toolchain.
1. Overview

To use Verifiable C, one must have had some experience using Coq, and some familiarity with the basic principles of Hoare logic. These can be obtained by studying Pierce’s *Software Foundations* interactive textbook, and doing the exercises all the way to chapter “Hoare2.”

It is also useful to read the brief introductions to Hoare Logic and Separation Logic, covered in Appel’s *Program Logics for Certified Compilers*, Chapters 2 and 3.

*Program Logics for Certified Compilers* (Cambridge University Press, 2014) describes Verifiable C version 1.1. If you are interested in the semantic model, soundness proof, or memory model of VST, the book is well worth reading. But it is not a reference manual.

More recent VST versions differ in several ways from what the PLCC book describes. • In the LOCAL component of an assertion, one writes temp \( i \ v \) instead of `\((\text{eq } v) (\text{eval} \_\text{id } i)\)`. • In the SEP component of an assertion, backticks are not used (predicates are not lifted). • In general, the backtick notation is rarely needed. • The type-checker now has a more refined view of char and short types. • field\_mapsto is now called field\_at, and it is dependently typed. • typed\_mapsto is renamed to data\_at, and last two arguments are swapped. • umapsto (“untyped mapsto”) no longer exists. • mapsto \( sh \ t \ v \ w \) now permits either \( (w = \text{Vundef}) \) or the value \( w \) belongs to type \( t \). This permits describing uninitialized locations, i.e., mapsto\_sh \( t \ v \) = mapsto\_sh \( t \ v \) \text{Vundef}. For function calls, one uses forward\_call instead of forward. • C functions may fall through the end of the function body, and this is (per the C semantics) equivalent to a return; statement.
2 Installation

The Verified Software Toolchain runs on Linux, Mac, or Windows. You will need to install:

1. Coq 8.4pl6, from coq.inria.fr. Follow the standard installation instructions.
2. CompCert 2.6, from compcert.inria.fr. You will want to build the clightgen tool, using these commands: ./configure ia32-linux; make clightgen. You might replace ia32-linux with ia32-macosx or ia32-cygwin. Verifiable C should work on other 32-bit architectures as well, but has not been extensively tested.
3. VST 1.6, from vst.cs.princeton.edu, or else an appropriate version from .

WORKFLOW. Within vst, the progs directory contains some sample C programs with their verifications. The workflow is:

- Write a C program \( F.c \).
- Run clightgen \( F.c \) to translate it into a Coq file \( F.v \).
- Write a verification of \( F.v \) in a file such as verif\( F.v \). That latter file must import both \( F.v \) and the VST Floyd\(^1\) program verification system, floyd.proofauto.

LOAD PATHS. Interactive development environments (CoqIDE or Proof General) will need their load paths properly initialized through command-line arguments. Running make in vst creates a file .loadpath with the right arguments. You can then do (for example),

cqide `cat .loadpath` progs/verif_reverse.v

See the heading USING PROOF GENERAL AND COQIDE in the file BUILD_ORGANIZATION for more information.

\(^1\)Named after Robert W. Floyd (1936–2001), a pioneer in program verification.
Verifiable C is a language (subset of C) and a program logic (higher-order impredicative concurrent separation logic).

In writing Verifiable C programs you must:

- Make each memory dereference into a top level expression (PLCC page 143)
- Avoid casting between integers and pointers.
- Avoid goto and switch statements.
  * Avoid nesting function calls and assignments inside subexpressions.
  * Factor && and || operators into if statements (to capture short circuiting behavior).

The items marked * are accomplished automatically by CompCert’s clightgen tool. That is, if you have function calls or assignments inside expressions, clightgen will factor the your program adding extra assignments to temporary variables.

There’s a special treatment of malloc/free; see Chapter 46.
4 Clightgen and ASTs

We will introduce Verifiable C by explaining the proof of a simple C program: adding up the elements of an array.

```c
#include <stddef.h>

int sumarray(int a[], int n) {
    int i, s, x;
    i = 0;
    s = 0;
    while (i < n) {
        x = a[i];
        s += x;
        i++;
    }
    return s;
}

int four[4] = {1, 2, 3, 4};

int main(void) {
    int s;
    s = sumarray(four, 4);
    return s;
}
```

You can examine this program in VST/progs/sumarray.c. Then look at progs/sumarray.v to find the output of CompCert’s `clightgen` utility: it is the abstract syntax tree (AST) of the C program, expressed in Coq. In sumarray.v there are definitions such as,

...  
**Definition** main : ident := 54%positive.
...
**Definition** s : ident := 50%positive.
... 

**Definition** \( f_{\text{sumarray}} := \{|\)

\[
\begin{align*}
\text{fn\_return} & := \text{tint}; \\
\text{fn\_params} & := ((-a, (\text{tptr tint})) :: (-n, \text{tint}) :: \text{nil}); \\
\text{fn\_temps} & := ((-i, \text{tint}) :: (-s, \text{tint}) :: (-x, \text{tint}) :: \text{nil}); \\
\text{fn\_body} & := \\
(\text{Ssequence} \\
(\text{Sset \_i (Econst\_int (Int\_repr 0) \text{tint})}) \\
(\text{Ssequence} \\
(\text{Sset \_s (Econst\_int (Int\_repr 0) \text{tint})}) \\
(\text{Ssequence} \ldots \\
)))) \\
\}|.
\]

...

**Definition** \( \text{prog} : \text{Clight\_program} := \{| \ldots |\}

In general it’s never necessary to read the AST file such as \text{sumarray.v}. But it’s useful to know what kind of thing is in there. C-language identifiers such as \text{main} and \text{s} are represented in ASTs as positive numbers; the definitions \text{\_main} and \text{\_s} are abbreviations for these. The AST for \text{sumarray} is in the function-definition \( f_{\text{sumarray}} \).

There you can see that \text{sumarray}’s return type is \text{is int}. To represent the syntax of C type-expressions, CompCert defines,

**Inductive** \( \text{type} : \text{Type} := \\
| \text{Tvoid}: \text{type} \\
| \text{Tint}: \text{intsize} \rightarrow \text{signedness} \rightarrow \text{attr} \rightarrow \text{type} \\
| \text{Tpointer}: \text{type} \rightarrow \text{attr} \rightarrow \text{type} \\
| \text{Tstruct}: \text{ident} \rightarrow \text{attr} \rightarrow \text{type} \\
| \ldots . \\

and we abbreviate \( \text{tint} := \text{Tint I32 Signed noattr} \).
5 Use the IDE

Chapter 6 through Chapter 16 are meant to be read while you have the file progs/verif_sumarray.v open in a window of your interactive development environment for Coq. You can use Proof General, CoqIDE, or any other IDE that supports Coq.

Reading these chapters will be much less informative if you cannot see the proof state as each chapter discusses it.

Before starting the IDE, read about load paths, at the heading USING PROOF GENERAL AND COQIDE in the file VST/BUILD_ORGANIZATION.
6 Functional spec, API spec

A program without a specification cannot be incorrect, it can only be surprising. (Paraphrase of J. J. Horning, 1982)

The file progs/verif_sumarray.v contains the specification of sumarray.c, and the proof of correctness of the C program with respect to that specification. For larger programs, one would typically break this down into three or more files:

1. Functional specification
2. API specification
3. Function-body correctness proofs, one per file.

To prove correctness of sumarray.c, we start by writing a functional spec of adding-up-a-sequence, then an API spec of adding-up-an-array-in-C.

FUNCTIONAL SPEC. A mathematical model of this program is the sum of a sequence of integers: $\sum_{i=0}^{n-1} x_i$. It’s conventional in Coq to use list to represent a sequence; we can represent the sum with a list-fold:

Definition sum_Z : list Z → Z := fold_right Z.add 0.

A functional spec contains not only definitions; it’s also useful to include theorems about this mathematical domain:

Lemma sum_Z_app: ∀ a b, sum_Z (a++b) = sum_Z a + sum_Z b.
Proof.
  intros. induction a; simpl; omega.
Qed.

The data types used in a functional spec can be any kind of mathematics at all, as long as we have a way to relate them to the integers, tuples, and sequences used in a C program. But the mathematical integers $\mathbb{Z}$ and the 32-bit modular integers $\mathbb{Int}.int$ are often relevant. Notice that this functional spec does not depend on sumarray.v or even on anything in the
Verifiable C libraries. This is typical, and desirable: the functional spec is about mathematics, not about C programming.

The Application Programmer Interface of a C program is expressed in its header file: function prototypes and data-structure definitions that explain how to call upon the modules’ functionality. In Verifiable C, an API specification is written as a series of function specifications (funspecs) corresponding to the function prototypes.

We start verif_sumarray.v with some standard boilerplate:

```plaintext
Require Import floyd.proofauto.
Require Import progs.sumarray.
Instance CompSpecs : comspecs. make_comspecs prog. Defined.
Definition Vprog : varspecs. mk_varspecs prog. Defined.
```

The first line imports Verifiable C and its Floyd proof-automation library. The second line imports the AST of the program to be proved. Lines 3 and 4 are identical in any verification: see Chapter 23 and Chapter 42.

After the boilerplate (and the functional spec), we have the function specifications for each function in the API spec:

```plaintext
Definition sumarray_spec :=
DECLARE _sumarray
  WITH a: val, sh : share, contents : list Z, size: Z
  PRE [ _a OF (tptr tint), _n OF tint ]
    PROP(readable_share sh;
      0 ≤ size ≤ Int.max_signed;
      Forall (fun x ⇒ Int.min_signed ≤ x ≤ Int.max_signed) contents)
    LOCAL(temp _a a; temp _n (Vint (Int.repr size)))
  SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a)
  POST [ tint ]
    PROP()
    LOCAL(temp ret_temp (Vint (Int.repr (sum-Z contents))))
    SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a).
```
The funspec begins, **Definition** \( f\text{-spec} := \text{DECLARE id}_f \ldots \) where \( f \) is the name of the C function.

A function is specified by its **precondition** and its **postcondition**. The **WITH** clause quantifies over Coq values that may appear in both the precondition and the postcondition. The precondition is parameterized by the C-language function parameters, and the postcondition is parameterized by a identifier \( \text{ret\_temp} \), which is short for, “the temporary variable holding the return value.” But really, the Coq variable \( _a \) does not have type (pointer-to-int); it has type \( \text{ident} \) (see page 8).

An assertion in Verifiable C’s **separation logic** can be written at either of two levels: The **lifted level**, implicitly quantifying over all local-variable states; or the **base level**, at a particular local-variable state. Program assertions are written at the lifted level, for which the notation is \( \text{PROP}(\ldots) \text{LOCAL}(\ldots) \text{SEP}(\ldots) \).

In an assertion \( \text{PROP}(\vec{P}) \text{LOCAL}(\vec{Q}) \text{SEP}(\vec{R}) \), the propositions in the sequence \( \vec{P} \) are all of Coq type Prop. They describe things that are forever true, independent of program state. Of course, in the function precondition above, the statement \( 0 \leq \text{size} \leq \text{Int.max\_signed} \) is “forever” true just within the scope of the quantification of the variable size; it is bound by **WITH** and spans the PRE and POST assertions.

The **LOCAL** propositions \( \vec{Q} \) are **variable bindings** of type \( \text{localdef} \). Here, the function-parameters \( a \) and \( n \) are treated as nonaddressable local variables, or “temp” variables. The localdef \( (\text{temp} _a a) \) says that (in this program state) the contents of C local variable \( _a \) is the Coq value \( a \). In general, the contents of a C scalar variable is always a \( \text{val} \); this type is defined by CompCert as,

\[
\text{Inductive val: Type := Vundef: val | Vint: int \rightarrow val | Vlong: int64 \rightarrow val | Vfloat: float \rightarrow val | Vsingle: float32 \rightarrow val | Vptr: block \rightarrow int \rightarrow val.}
\]

The **SEP** conjuncts \( \vec{R} \) are **spatial assertions** in separation logic. In this
case, there’s just one, a data_at assertion saying that at address a in memory, there is a data structure of type array[size] of integers, with access-permission sh, and the contents of that array is the sequence map Vint contents.

The postcondition is introduced by POST [ tint ], indicating that this function returns a value of type int. There are no PROP statements in the postcondition, because no forever-true facts exist in the world that weren’t already true on entry to the function. (This is typical!) The LOCAL must not mention the function parameters, because they are destroyed on function exit; it will only mention the return-temporary ret_temp. The SEP clause mentions all the spatial resources from the precondition, minus ones that have been freed (deallocated), plus ones that have been malloc’d (allocated).

So, overall, the specification for sumarray is this: “At any call to sumarray, there exist values a,sh,contents,size such that sh gives at least read-permission; size is representable as a nonnegative 32-bit signed integer; function-parameter .a contains value a and .n contains the 32-bit representation of size; and there’s an array in memory at address a with permission sh containing contents. The function returns a value equal to sum_int(contents), and leaves the array unaltered.”

Integer overflow. The C language specification says that a C compiler may treat signed integer overflow by wrapping around mod \(2^n\), where n is the word size (e.g., 32). In practice, almost all C compilers (including CompCert) do this wraparound, and it is part of the CompCert C light operational semantics. See Chapter 20. The function Int.repr: \(Z \rightarrow \text{int}\) truncates mathematical integers into 32-bit integers by taking the (sign-extended) low-order 32 bits. Int.signed: \(\text{int} \rightarrow Z\) injects back into the signed integers.

The postcondition guarantees that the value return is Int.repr (sum_Z contents). But what if \(\sum s \geq 2^{31}\), so the sum doesn’t fit in a 32-bit signed integer? Then Int.signed(Int.repr (sum_Z contents)) \(\neq\) (sum_Z contents). In gen-
eral, for a claim about $\text{Int}.\text{repr}(x)$ to be *useful*, one also needs a claim that $0 \leq x \leq \text{Int}.\text{max}_\text{unsigned} \text{ or } \text{Int}.\text{min}_\text{signed} \leq x \leq \text{Int}.\text{max}_\text{signed}$. The caller of this function will probably need to prove $\text{Int}.\text{min}_\text{signed} \leq \text{sum}_\mathbb{Z} \text{ contents} \leq \text{Int}.\text{max}_\text{signed}$ in order to make much use of the post-condition.

What if $s$ is the sequence $[\text{Int}.\text{max}_\text{signed}; 5; 1-\text{Int}.\text{max}_\text{signed}]$? Then $\sum s = 6$. Does the program really work? Answer: Yes, by the miracle of modular arithmetic.
7 Proof of the sumarray program

To prove correctness of a whole program,

1. Collect the function-API specs together into Gprog: list funspec.
2. Prove that each function satisfies its own API spec (with a semax.body proof).
3. Tie everything together with a semax.func proof.

In progs/verif_sumarray.v, the first step is easy:

**Definition** Gprog : funspecs := sumarray_spec :: main_spec::nil.

The function specs, built using DECLARE, are listed in the same order the functions appear in the program (in particular, the same order they appear in prog.(prog.defs), in sumarray.v).

In addition to Gprog, the API spec contains Vprog, the list of global-variable type-specs. This is computed automatically by the mk_varspecs tactic, as shown at the beginning of verif_sumarray.v.

Each C function can call any of the other C functions in the API, so each semax.body proof is a client of the entire API spec, that is, Vprog and Gprog. You can see that in the statement of the semax.body lemma for the _sumarray function:

**Lemma** body_sumarray: semax.body Vprog Gprog f_sumarray sumarray_spec.

Here, f_sumarray is the actual function body (AST of the C code) as parsed by clightgen; you can read it in sumarray.v. You can read body_sumarray as saying, *In the context of Vprog and Gprog, the function body f_sumarray satisfies its specification sumarray_spec.* We need the context in case the sumarray function refers to a global variable (Vprog provides the variable’s type) or calls a global function (Gprog provides the function’s API spec).
8 start_function

The predicate semax_body states the Hoare triple of the function body, \( \Delta \vdash \{ \text{Pre} \} c \{ \text{Post} \} \). \text{Pre} and \text{Post} are taken from the funspec for \( f \), \( c \) is the body of \( F \), and the type-context \( \Delta \) is calculated from the global type-context overlaid with the parameter- and local-types of the function.

To prove this, we begin with the tactic start_function, which takes care of some simple bookkeeping and expresses the Hoare triple to be proved.

**Lemma** body_sumarray: semax_body Vprog Gprog f_sumarray sumarray_spec.

**Proof.**

start_function.

The proof goal now looks like this:

```
Espec : OracleKind
a : val
sh : share
contents : list Z
size : Z
Delta_specs := abbreviate : PTree.t funspec
Delta := abbreviate : tycontext
SH : readable_share sh
H : 0 \leq size \leq \text{Int.max_signed}
H0 : \forall x : Z \Rightarrow \text{Int.min_signed} \leq x \leq \text{Int.max_signed} \quad \text{contents}
POSTCONDITION := abbreviate : ret_assert
MORE_COMMANDS := abbreviate : statement
```

```
--------------------------------------------------------------------(1/1)
semax Delta
  \text{PROP} ()
  \text{LOCAL} (temp_a a; temp_n (Vint (Int.repr size)))
  \text{SEP} (data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a))
(Ssequence (Sset_i (Econst_int (Int.repr 0) tint)) MORE_COMMANDS)
POSTCONDITION
```
First we have *Espec*, which you can ignore for now (it characterizes the outside world, but sumarray.c does not do any I/O). Then `a,sh,contents,size` are exactly the variables of the WITH clause of `sumarray_spec`.

The two abbreviations `Delta_spec, Delta` are the type-context in which Floyd’s proof tactics will look up information about the types of the program’s variables and functions. The hypotheses `SH,H,H0` are exactly the PROP clause of `sumarray_spec`’s precondition. The POSTCONDITION is exactly the POST part of `sumarray_spec`.

To see the contents of an abbreviation, either (1) set your IDE to show implicit arguments, or (2) (e.g.,) unfold abbreviate in POSTCONDITION.

Below the line we have one proof goal: the Hoare triple of the function body. In this judgment `Δ ⊢ {P} c {R}`, written in Coq as `semax (Δ : tycontext) (P : environ→mpred) (c : statement) (R : ret_assert)`

\[ \Delta \text{ is a type context, giving types of function parameters, local variables, and specifications (funspec) of global functions.} \]
\[ P \text{ is the precondition;} \]
\[ c \text{ is a command in the C language; and} \]
\[ R \text{ is the postcondition. Because a } c \text{ statement can exit in different ways (fall-through, continue, break, return), a ret_assert has predicates for all of these cases.} \]

Because we do forward Hoare-logic proof, we won’t care about the postcondition until we get to the end of `c`, so here we hide it away in an abbreviation. Here, the command `c` is a long sequence starting with `i=0; ...more`, and we hide the `more` in an abbreviation `MORE_COMMANDS`.

The precondition of this `semax` has LOCAL and SEP parts taken directly from the funspec (the PROP clauses have been moved above the line). The statement `(Sset .i (Econst_int (Int.repr 0) tint))` is the AST generated by clightgen from the C statement `i=0;`. 
We do Hoare logic proof by forward symbolic execution. On page 17 we show the proof goal at the beginning of the sumarray function body. In a forward Hoare logic proof of $\{P\} i = 0; more \{R\}$ we might first apply the sequence rule,

$$
\begin{align*}
\{P\} i = 0 \{Q\} & \quad \{Q\} more \{R\} \\
\{P\} i = 0; more \{R\} & 
\end{align*}
$$

assuming we could derive some appropriate assertion $Q$.

For many kinds of statements (assignments, return, break, continue) this is done automatically by the forward tactic. When we execute forward here, the resulting proof goal is,

Espec, a, sh, contents, size, Delta_spec, SH, H, H0 as before
Delta := abbreviate : tycontext
POSTCONDITION := abbreviate : ret_assert
MORE_COMMANDS := abbreviate : statement

---------------------------------------------------------------------(1/1)

semax Delta
(PROP ()

LOCAL(temp .i (Vint (Int.repr 0)); temp .a a;
  temp .n (Vint (Int.repr size)))
  SEP(data.at sh (tarray tint size) (map Vint (map Int.repr contents)) a))
(Ssequence (Sset .s (Econst_int (Int.repr 0) tint)) MORE_COMMANDS)
POSTCONDITION

Notice that the precondition of this semax is really the postcondition of the $i=0$; statement; it is the precondition of the next statement, $s=0;$. It’s much like the precondition of $i=0$; what has changed?

- The LOCAL part contains temp .i (Vint (Int.repr 0)) in addition to what it had before; this says that the local variable $i$ contains integer value zero.
• the command is now \texttt{s=0;more}, where MORE_COMMANDS no longer contains \texttt{s=0;}.

• Delta has changed; it now records the information that \(i\) is initialized.

Another forward goes through \texttt{s=0;} to yield a proof goal with a LOCAL binding for the \_s variable.

\textbf{FORWARD} \textit{works on several kinds of C commands.} In each of the following cases, the expression \(E\) must not contain side effects or function calls. The variable \(x\) must be a nonaddressable local variable.

\(c_1;\ c_2\) Sequencing of two commands. The forward tactic will work on \(c_1\) first.

\((c_1;\ c_2)\ c_3\) In this case, forward will re-associate the commands using the \texttt{seq_assoc} axiom, and work on \(c_1;\ (c_2;\ c_3)\).

\(x=E;\) Assignment statement. Expression \(E\) must not contain memory dereferences (loads or stores using \texttt{*}prefix, suffix[], or \texttt{-} operators). No restrictions on the form of the precondition (except that it must be in canonical form). The expression \&\texttt{p}→\texttt{next} does not actually load or store (it just computes an address) and is permitted.

\(x=\texttt{*}E;\) Memory load.

\(x=\texttt{a[E]};\) Array load.

\(x=\texttt{E→fld};\) Field load.

\(x=\texttt{E→f_1.f_2};\) Nested field load.

\(x=\texttt{E→f_1[i].f_2};\) Fields and subscripts …When the right-hand side is equivalent to a single memory-load via some access \textit{path} (struct-fields and array-subscripts) from pointer value \(p\), the SEP component of the precondition must contain an appropriately typed item of the form \texttt{data_at \pi t v p} such that the \textit{path} from \(p\) in an object of type \(t\) leads to a field (or array slot) that can be loaded into \_x. Or, \texttt{field_at \pi t path' v p'}, such that where \texttt{path'} is a suffix of \textit{path}, and \(p'\) is the address reached by starting at \(p\) and following the prefix. Share \(\pi\) must be a readable_share.
\( E_1 = E_2; \) Memory store. Expression \( E_2 \) must not dereference memory. Expression \( E_1 \) must be equivalent to a single memory store via some access path (as described above for loads), and there must be an appropriate storable data at or field at. Or \( E_1 \) may be an addressable local variable. Share \( \pi \) must be a writable share.

**if \((E)\) \(C_1\) else \(C_2\)** For an if-statement, use \texttt{forward-if} and provide a postcondition.

**while \((E)\) \(C\)** For a while-loop, use the \texttt{forward-while} tactic (page 22) and provide a loop invariant.

**break;** The forward tactic works.

**continue;** The forward tactic works.

**return \(E\);** Expression \(E\) must not dereference memory, and the presence/absence of \(E\) must match the nonvoid/void return type of the function. The proof goal left by forward is to show that the precondition (with appropriate substitution for the abstract variable \texttt{ret\_var}) entails the function's postcondition.

\( x = f(a_1, \ldots, a_n); \)** For a function call, use \texttt{forward\_call(W)}, where \(W\) is a witness, a tuple corresponding (componentwise) to the \texttt{WITH} clause of the function specification. (If you do just forward, you'll get a message with advice about the type of \(W\).)

This results a proof goal to show that the precondition implies the function precondition and includes an uninstantiated variable: The Frame represents the part of the spacial precondition that is unchanged by the function call. It will generally be instantiated by a call to cancel.
10 While loops

To prove a while loop by forward symbolic execution, you use the tactic forward-while, and you must supply a loop invariant. Take the example of the forward-while in progs/verif_sumarray.v. The proof goal is,

Espec, Delta_specs, Delta
a : val, sh : share, contents : list Z, size : Z
SH : readable_share sh
H : 0 ≤ size ≤ Int.max_signed
H0 : Forall (fun x : Z ⇒ Int.min_signed ≤ x ≤ Int.max_signed) contents
POSTCONDITION := abbreviate : ret_assert
MORE_COMMANDS, LOOP_BODY := abbreviate : statement
----------------------------------------------------------------(1/1)
semax Delta
  (PROP ()
    LOCAL(temp _s (Vint (Int.repr 0)); temp _i (Vint (Int.repr 0));
       temp _a a; temp _n (Vint (Int.repr size)))
    SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a))
  (Ssequence
     (Swhile (Ebinop Olt (Etempvar _i tint) (Etempvar _n tint) tint)
        LOOP_BODY)
     MORE_COMMANDS)
  POSTCONDITION

A loop invariant is an assertion, almost always in the form of an existential EX...PROP()LOCAL()SEP(). Each iteration of the loop has a state characterized by a different value of some iteration variable(s), the the EX binds that value. For example, the invariant for this loop is,

Definition sumarray_Inv a0 sh contents size :=
EX i : Z,
  PROP(0 ≤ i ≤ size)
  LOCAL(temp _a a0; temp _i (Vint (Int.repr i)); temp _n (Vint (Int.repr size));
     temp _s (Vint (Int.repr (sum_Z (sublist 0 i contents)))))
  SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a0).
The existential binds $i$, the iteration-dependent value of the local variable named `_i`. In general, there may be any number of EX quantifiers.

1. the precondition (of the whole loop) implies the loop invariant;
2. the loop-condition expression type-checks (i.e., guarantees to evaluate successfully);
3. the postcondition of the loop body implies the loop invariant;
4. the loop invariant (and not loop condition) is a good precondition for the proof of the MORE_COMMANDS after the loop.

Let’s take a look at that first subgoal:

\[
\begin{aligned}
\text{ENTAIL Delta,} \\
\quad \text{PROP}() \\
\quad \text{LOCAL} (\text{temp }_s (\text{Vint (Int.repr 0)}); \text{temp }_i (\text{Vint (Int.repr 0)})); \\
\quad \text{temp }_a a; \text{temp }_n (\text{Vint (Int.repr size)})) \\
\quad \text{SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a}) \\
\vdash \text{EX } i : Z, \\
\quad \text{PROP}(0 \leq i \leq \text{size}) \\
\quad \text{LOCAL} (\text{temp }_a a; \text{temp }_i (\text{Vint (Int.repr } i)); \\
\quad \text{temp }_n (\text{Vint (Int.repr size)}); \\
\quad \text{temp }_s (\text{Vint (Int.repr (sum_Z (sublist 0 } i \text{ contents}))})) \\
\quad \text{SEP(data_at sh (tarray tint size) (map Vint (map Int.repr contents)) a})
\end{aligned}
\]

This is an entailment goal; Chapter 11 shows how to prove such goals.
11 Entailments

An entailment in separation logic, $P \vdash Q$, says that any state satisfying $P$ must also satisfy $Q$. What’s in a state? Local-variable environment, heap (addressable memory), even the state of the outside world. VST's type `mpred`, memory predicate, can be thought of as `mem\rightarrow\text{Prop}` (but is not quite the same, for quite technical semantic reasons). That is, an `mpred` is a test on the heap only, and cannot “see” the local variables (tempvars) of the C program.

Type `environ` is a local/global variable environment, mapping identifiers (ident) to the values of globals, addressable locals, and tempvars (nonaddressable locals). A lifted predicate of type `environ\rightarrow mpred` can “see” both the heap and the local/global variables. The Pre/Post arguments of Hoare triples (semax $\Delta\,\text{Pre}\,c\,\text{Post}$) are lifted predicates.

At present, Verifiable C has a notion of external-world state, in the Espec: OracleKind, but it is not well developed; enhancements will be needed for reasoning about input/output.

Our language for lifted predicates uses $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$, where $\vec{R}$ is a list of `mpreds`. Our language for `mpreds` uses primitives such as `data\_at` and `emp`, along with connectives such as the `$\ast$` and `$\neg\ast$` of separation logic. In both languages there is an `EX` operator for existential quantification.

Separation logic’s rule of consequence is shown here

\[
\frac{P \vdash P' \quad \{P'\} \text{c} \{Q'\} \quad Q' \vdash Q}{\{P'\} \text{c} \{Q'\}} \quad \frac{\Delta, P \vdash P' \quad \text{semax } \Delta\,P'\,c\,Q' \quad \Delta, Q' \vdash Q}{\text{semax } \Delta\,P\,c\,Q}
\]

at left in traditional notation, and at right as in Verifiable C. The type-context $\Delta$ constrains values of locals and globals. Using this axiom, called `semax\_pre\_post` on a proof goal `semax $\Delta\,P\,c\,Q$` yields three subgoals: another `semax` and two (lifted) entailments, $\Delta, P \vdash P'$ and $\Delta, Q \vdash Q'$. 
The standard form of a lifted entailment is $\text{ENTAIL } \Delta, PQR \vdash PQR'$, where $PQR$ and $PQR'$ are typically in the form $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$, perhaps with some EX quantifiers in the front. The turnstile $\vdash$ is written in Coq as $\mid\--\mid$.

Let's consider the entailment arising from forward\_while in the progs/verif\_sumarray example:

$H : 0 \leq \text{size} \leq \text{Int.max\_signed}$

\[\text{(other above-the-line hypotheses elided)}\]

$\text{ENTAIL } \Delta,\$

\begin{align*}
\text{PROP}( & \
\text{LOCAL}(\text{temp \_s (Vint (Int.repr 0)); temp \_i (Vint (Int.repr 0))}; \text{temp \_a a; temp \_n (Vint (Int.repr size))}) & \\
\text{SEP(} & \text{data\_at sh (tarray tint size) (map Vint (map Int.repr contents)) a)}
\end{align*}

$\vdash \text{EX } i : Z,$

\begin{align*}
\text{PROP}(0 \leq i \leq & \text{size}) & \\
\text{LOCAL}( & \text{temp \_a a; temp \_i (Vint (Int.repr i))}; \text{temp \_n (Vint (Int.repr size))}; \text{temp \_s (Vint (Int.repr (sum\_Z (sublist 0 i contents)))))}) & \\
\text{SEP(} & \text{data\_at sh (tarray tint size) (map Vint (map Int.repr contents)) a)}
\end{align*}

We instantiate the existential with the only value that works here, zero: $\text{Exists } 0$. Chapter 19 explains how to handle existentials with $\text{Intros}$ and $\text{Exists}$.

Now we use the entailer! tactic to solve as much of this goal as possible (see Chapter 35). In this case, the goal solves entirely automatically. In particular, $0 \leq i \leq \text{size}$ solves by omega; sublist 0 0 contents rewrites to nil; and $\text{sum\_Z nil}$ simplifies to 0.

The second subgoal of forward\_while in progs/verif\_sumarray.v is a type-checking entailment, of the form $\text{ENTAIL } \Delta, PQR \vdash \text{tc\_expr } \Delta e$ where $e$ is (the abstract syntax of) a C expression; in the particular case of a while loop, $e$ is the negation of the loop-test expression. The
entailment guarantees that \( e \) executes without crashing: all the variables it references exist, and are initialized; and it doesn’t divide by zero, et cetera.

In this case, the entailment concerns the expression \( \neg(i < n) \),

\[
\text{ENTAIL \ Delta, PROP(...) LOCAL(...) SEP(....)}
\]
\[
\vdash \text{tc-expr Delta}
\]
\[
(Eunop Onotbool (Ebinop Olt (Etempvar \_i tint) (Etempvar \_n tint) tint) tint)
\]

This solves completely via the entail! tactic. To see why that is, instead of doing entail!, do unfold tc_expr; simpl. You’ll see that the right-hand side of the entailment simplifies down to \(!!\text{True}\). That’s because the typechecker is *calculational*, as Chapter 25 of *Program Logics for Certified Compilers* explains.
12 Array subscripts

The third subgoal of forward_while in progs/verif_sumarray.v is the body of the while loop: \{x=a[i]; s+=x; i++;\}.

This can be handled by three forward commands, but the first one of these leaves a subgoal—proving that the subscript \(i\) in in range. Let’s examine the proof goal:

\[
\begin{align*}
SH : & \text{readable\_share } sh \\
H : & 0 \leq size \leq \text{Int}\_\text{max\_signed} \\
H0 : & \text{Forall } (\text{fun } x : Z \Rightarrow \text{Int}\_\text{min\_signed} \leq x \leq \text{Int}\_\text{max\_signed}) \text{ contents} \\
i : & Z \\
HRE : & i < size \\
H1 : & 0 \leq i \leq size
\end{align*}
\]

semax Delta

\[
\begin{align*}
(\text{PROP } () & ) \\
& \text{LOCAL}(\text{temp }_a a; \text{temp }_i (\text{Vint (Int}\_\text{repr } i)); \\
& \text{temp }_n (\text{Vint (Int}\_\text{repr size)})) ; \\
& \text{temp }_s (\text{Vint (Int}\_\text{repr } (\text{sum}\_Z (\text{sublist } 0 i \text{ contents})))) \\
& \text{SEP(data}\_\text{at } sh (\text{tarray } tint \text{ size}) (\text{map Vint (map Int}\_\text{repr contents}) a)) \\
& (\text{Ssequence} \\
& (\text{Sset }_x \\
& (\text{Ederef} \\
& (\text{Ebinop Oadd (Etempvar }_a (\text{tptr } tint)) (\text{Etempvar }_i tint) \\
& (\text{tptr } tint)) tint))) \text{ MORE\_COMMANDS) POSTCONDITION}
\end{align*}
\]

The Coq variable \(i\) was introduced automatically by forward_while from the existential variable, the EX \(i:Z\) of the loop invariant.

The command \(x=a[i]\) is a load from data-struture \(a\). For this to succeed, there must be a data_at (or field_at) assertion about \(a\) in the SEP clauses of the precondition; the permission share in that data_at must grant read access; and the subscript must be in range. Indeed, the data_at is there,
and the share is taken care of automatically by the hypothesis \( \text{SH} \) above the line.

So, forward succeeds; but it leaves an array-bounds subgoal:

\[
\text{ENTAIL Delta, PROP(...) LOCAL(...) SEP(...)}
\]
\[
\vdash \text{tc_expr Delta (Etempvar_a (tptr tint)) \&\&}
\]
\[
\quad \text{local `(tc_val tint (Znth i (map Vint (map Int.repr contents)) Vundef)) \&\&}
\]
\[
\quad (\text{tc_expr Delta (Etempvar_i tint) \&\& TT})
\]

The two \( \text{tc_expr} \) conjuncts are trivial (they are \( \beta\eta \)-equal to \( \text{TT} \)) but the middle conjunct is nontrivial. To clean things up, we run entailer!, which leaves this subgoal:

\[
\text{HRE : i < Zlength (map Vint (map Int.repr contents))}
\]
\[
\text{H1 : 0 \leq i \leq Zlength (map Vint (map Int.repr contents))}
\]
\[
\quad (\text{other above-the-line hypotheses elided})
\]
\[
\text{is_int I32 Signed (Znth i (map Vint (map Int.repr contents)) Vundef)}
\]

For the load to succeed, the \( i \) element of \( \text{(map Vint (map Int.repr contents))} \) must actually be an integer, not an undefined value. To prove this, we use the \( \text{Znth_map} \) lemma to move the \( \text{Znth} \) inside the \( \text{Vint} \), leaving the goal,

\[
\text{is_int I32 Signed (Vint (Znth i (map Vint (map Int.repr contents)) Int.zero))}
\]

This is an instance of \( \text{is_int I32 Signed (Vint ...)} \) which is \( \beta\eta \)-equal to True. However, when we rewrote by \( \text{Znth_map} \), that left a subgoal,

\[
\text{HRE : i < Zlength (map Vint (map Int.repr contents))}
\]
\[
\text{H1 : 0 \leq i \leq Zlength (map Vint (map Int.repr contents))}
\]
\[
\quad (\text{other above-the-line hypotheses elided})
\]
\[
0 \leq i < \text{Zlength (map Int.repr contents)}
\]

This solves straightforwardly as shown in the proof script.
13 Splitting sublists

In progs/verif_sumarray.v, at the comment “Now we have reached the end of the loop body,” it is time to prove that the current precondition (which is the postcondition of the loop body) entails the loop invariant. This is the proof goal:

\[ \begin{align*}
H & : 0 \leq \text{size} \leq \text{Int.max_signed} \\
H0 & : \text{Forall } (\text{fun } x : \text{Z} \Rightarrow \text{Int.min_signed} \leq x \leq \text{Int.max_signed}) \text{ contents} \\
HRE & : i < \text{size} \\
H1 & : 0 \leq i \leq \text{size} \\
\text{(other above-the-line hypotheses elided)}
\end{align*} \]

\[ \text{ENTAIL Delta,} \]
\[ \text{PROP()} \]
\[ \text{LOCAL(} \text{temp } _i (\text{Vint (} \text{Int.add (} \text{Int.repr } i \text{) (} \text{Int.repr 1)})))); \]
\[ \text{temp } _s \]
\[ \text{(} \text{force_val} \]
\[ \text{,(} \text{sem.add.default tint tint} \]
\[ \text{(} \text{Vint (} \text{Int.repr (} \text{sum.Z (sublist 0 } i \text{ contents)})\))) \]
\[ \text{(} \text{Znth } i \text{ (} \text{map Vint (} \text{map Int.repr contents) Vundef)})\)); \]
\[ \text{temp } _x \text{ (} \text{Znth } i \text{ (} \text{map Vint (} \text{map Int.repr contents) Vundef)}; \text{temp } _a \text{ a; } \]
\[ \text{temp } _n \text{ (} \text{Vint (} \text{Int.repr size)})\)) \]
\[ \text{SEP(} \text{data.at sh (} \text{tarray tint size)} \text{ (} \text{map Vint (} \text{map Int.repr contents)}) \text{ a)} \]
\[ \vdash \text{EX } a_0 : \text{Z,} \]
\[ \text{PROP(}0 \leq a_0 \leq \text{size)} \]
\[ \text{LOCAL(} \text{temp } _a \text{ a; } \text{temp } _i \text{ (} \text{Vint (} \text{Int.repr } a_0)) ;} \]
\[ \text{temp } _n \text{ (} \text{Vint (} \text{Int.repr size)})\); \]
\[ \text{temp } _s \text{ (} \text{Vint (} \text{Int.repr (} \text{sum.Z (sublist 0 } a_0 \text{ contents)})\)))\)) \]
\[ \text{SEP(} \text{data.at sh (} \text{tarray tint size)} \text{ (} \text{map Vint (} \text{map Int.repr contents)}) \text{ a)} \]

The right-hand side of this entailment is just the loop invariant. As usual at the end of a loop body, there is an existentially quantified variable that must be instantiated with an iteration-dependent value. In this case it’s obvious: the quantified variable represents the contents of C local variable \( _i \), so we do, \text{Exists} \ (i+1).
The resulting entailment has many trivial parts and a nontrivial residue. The usual way to get to the hard part is to run entail!, which we do now. After clearing away the irrelevant hypotheses, we have:

\[
\begin{align*}
H & : 0 \leq Z\text{length} (\text{map Vint (map \text{Int}.\text{repr} \text{contents}))} \leq \text{Int}.\text{max}\_\text{signed} \\
\text{HRE} & : i < Z\text{length} (\text{map Vint (map \text{Int}.\text{repr} \text{contents}))} \\
\text{H1} & : 0 \leq i \leq Z\text{length} (\text{map Vint (map \text{Int}.\text{repr} \text{contents}))} \\
\end{align*}
\]

\[\text{(1/1)}\]

\[
\text{Vint} (\text{Int}.\text{repr} (\text{sum}_Z (\text{sublist} 0 (i + 1) \text{contents}))) = \text{force_val}
\]

\((\text{sem}_\text{add}\_\text{default} \text{tint} \text{tint} (\text{Vint} (\text{Int}.\text{repr} (\text{sum}_Z (\text{sublist} 0 \text{i contents}))))) \\
(Z\text{nth} \text{i} (\text{map Vint (map \text{Int}.\text{repr} \text{contents}))} \text{Vundef})\)

The \text{sem}_\text{add}\_\text{default} comes from the semantics of C expression evaluation: adding integers means one thing, but adding an integer to a Vundef is undefined, and so on. To clear that sludge out of the way, we move the \text{Znth} inside the Vint just as on page 28, then simpl, yielding this goal:

\[
\begin{align*}
H & : 0 \leq Z\text{length} \text{contents} \leq \text{Int}.\text{max}\_\text{signed} \\
\text{HRE} & : i < Z\text{length} \text{contents} \\
\text{H1} & : 0 \leq i \leq Z\text{length} \text{contents} \\
\end{align*}
\]

\[\text{(1/1)}\]

\[
\text{Vint} (\text{Int}.\text{repr} (\text{sum}_Z (\text{sublist} 0 (i + 1) \text{contents}))) = \\
\text{Vint} (\text{Int}.\text{add} (\text{Int}.\text{repr} (\text{sum}_Z (\text{sublist} 0 \text{i contents})))) \\
(\text{Int}.\text{repr} (Z\text{nth} \text{i} \text{contents} 0)))
\]

The lemma \text{add}\_\text{repr}: \forall i, j, \text{Int}.\text{add} (\text{Int}.\text{repr} i) (\text{Int}.\text{repr} j) = \text{Int}.\text{repr} (i + j) is useful here; followed by f\_equal, leaves:

\[
\begin{align*}
\text{sum}_Z (\text{sublist} 0 (i + 1) \text{contents}) = \\
\text{sum}_Z (\text{sublist} 0 \text{i contents}) + Z\text{nth} \text{i} \text{contents} 0
\end{align*}
\]

Now the lemma \text{sublist}\_\text{split}: \forall l, m, h, a_l, \ 0 \leq l \leq m \leq h \leq |a_l| \rightarrow \\
\text{sublist} l h a_l = \text{sublist} l m a_l ++ \text{sublist} m h a_l is helpful here:

rewrite (\text{sublist}\_\text{split} 0 \text{i} (i+1)) by omega. A bit more rewriting with the theory of \text{sum}_Z and sublist finishes the proof.
14 Returning from a function

In progs/verif_sumarray.v, at the comment “After the loop,” we have reached the return statement. The forward tactic works here, leaving a proof goal that the precondition of the return entails the postcondition of the function-spec. (When this automatically, it leaves no proof goal at all.) The goal is a lowered entailment (on mpred assertions).

After doing simpl to clear away some C-expression-evaluation sludge, we have

H4 : Forall (value_fits tint) (map Vint (map Int.repr contents))
H2 : field_compatible (Tarray tint (Zlength ...) noattr) [] a
   (other above-the-line hypotheses elided)

data_at sh (tarray tint (Zlength ...)) (map Vint (map Int.repr contents)) a
⊢ !(Vint (Int.repr (sum_Z contents)) =
   Vint (Int.repr (sum_Z (sublist 0 i contents))))

The left-hand side of this entailment is a spatial predicate (data_at). Purely nonspatial facts (H4 and H2) derivable from it have already been inferred and moved above the line by saturate_local (see Chapter 31).

This entailment’s right-hand side has no spatial predicates. That’s because the SEP clause of the funspec’s postcondition had exactly the same data_at clause as we see here in the entailment precondition, and the entailment-solver called by forward has already cleared it away.

In a situation like this—where saturate_local has already been done and the r.h.s. of the entailment is purely nonspatial—almost always there’s no more useful information in the left hand side that hasn’t already been extracted by saturate_local. We can throw away the l.h.s. with apply prop_right (or by entailer! but that’s a bit slower).

The remaining subgoal solves easily in the theory of sublists. The proof of the function sumarray is now complete.
C programs may have “extern” global variables, either with explicit initializers or initialized by default. Any function that accesses a global variable must have the appropriate spatial assertions in its funspec’s precondition (and postcondition). But the main function is special: it has spatial assertions for all the global variables. Then it may pass these on, piecemeal, to the functions it calls on an as-needed basis.

The function-spec for main always looks the same:

**Definition** main Spec :=

```plaintext
DECLARE _main WITH u : unit
   PRE [] main pre prog u
   POST [ tint ] main post prog u.
```

main pre calculates the precondition automatically from (the list of extern global variables and initializers of) the program. Then, when we prove that main satisfies its funspec,

**Lemma** body main: semax body Vprog Gprog f.main main Spec.

**Proof.**
name four _four.
start function.

the start function tactic “unpacks” main pre into an assertion:

```plaintext
four : val
--------------------------------------(1/1)
semax Delta
   (PROP () LOCAL(gvar _four four)
     SEP(data at Ews (tarray tint 4)
       (map Vint [Int repr 1; Int repr 2; Int repr 3; Int repr 4]) four))
   (... function body ...)
POSTCONDITION
```
The **LOCAL** clause means that the C global variable _four_ is at memory address *four*. (If we had omitted the name tactic in the proof script above, then `start-function` would have chosen some other name for this value.) See Chapter 29.

The **SEP** clause means that there’s data of type “array of 4 integers” at address *four*, with access permission Ews and contents [1;2;3;4]. Ews stands for “external write share,” the standard access permission of extern global writable variables. See Chapter 38.

Now it’s time to prove the function-call statement, `s = sumarray(four,4)`. When proving a function call, one must supply a *witness* for the **WITH** clause of the function-spec. The _sumarray_ function’s **WITH** clause binds variables `a:val, sh:share, contents:list Z, size: Z`, so the type of the witness will be `(val*(share*(list Z * list Z)))`. To choose the witness, examine your actual parameter values (along with the precondition of the funspec) to see what witness would be consistent; here, we use `(four,Ews,four_contents,4)`.

The forward-call tactic (usually) leaves subgoals: you must prove that your current precondition implies the funspec’s precondition. Here, these solve easily, as shown in the proof script.

The postcondition of the call statement (which is the precondition of the next return statement) has an existential, `EX vret:val`. This comes directly from the existential in the funspec’s postcondition. To move `vret` above the line, simply `Intros vret`.

Finally, we are at the return statement. The forward tactic is easily able to prove that the current assertion implies the postcondition of _main_, because `main_post` is basically an abbreviation for True.
16 Tying all the functions together

We build a whole-program proof by composing together the proofs of all the function bodies. Consider Gprog, the list of all the function specifications:

Definition Gprog : funspecs := sumarray_spec :: main_spec :: nil.

Each semax_body proof says, assuming that all the functions I might call behave as specified, then my own function-body indeed behaves as specified:

Lemma body_sumarray: semax_body Vprog Gprog f_sumarray sumarray_spec.

Note that all the functions I might call might even include “myself,” in the case of a recursive or mutually recursive function.

This might seem like circular reasoning, but it is actually sound—by the miracle of step-indexed semantic models, as explained in Chapters 18 and 39 of Program Logics for Certified Compilers.

The rule for tying the functions together is called semax_func, and its use is illustrated in this theorem, the main proof-of-correctness theorem for the program sumarray.c:

Lemma all_funcs_correct: semax_func Vprog Gprog (prog_funct prog) Gprog.
Proof.
unfold Gprog, prog, prog_funct; simpl.
semax_func_skipn.
semax_func_cons body_sumarray.
semax_func_cons body_main.
apply semax_func_nil.
Qed.

The calls to semax_func_cons must appear in the same order as the functions are listed in Gprog and the same order as they appear in prog.(prog_defs).
17 Separation logic: EX, *, emp, !!

The base level separation logic is built, like any separation logic, from predicates on “heaplets”. The grammar of base-level separation-logic expressions is,

\[
R ::= \text{emp} \quad \text{empty} \\
\text{TT} \quad \text{True} \\
\text{FF} \quad \text{False} \\
R_1 \ast R_2 \quad \text{separating conjunction} \\
R_1 \&\& R_2 \quad \text{ordinary conjunction} \\
\text{field}_{-}\text{at } \pi \tau \vec{f}ld v p \quad \text{“field maps-to”} \\
\text{data}_{-}\text{at } \pi \tau v p \quad \text{“maps-to”} \\
\text{array}_{-}\text{at } \tau \pi v lo hi \quad \text{array slice} \\
!!P \quad \text{pure proposition} \\
\text{EX } x : T, R \quad \text{existential quantification} \\
\text{ALL } x : T, R \quad \text{universal quantification (rare)} \\
R_1 \parallel R_2 \quad \text{disjunction} \\
\text{wand } R R' \quad \text{magic wand } R \rightarrow R' \text{ (rare)} \\
\ldots \quad \text{other operators, including user definitions}
\]
18 PROP( ) LOCAL( ) SEP( )

The *lifted* separation logic can “see” local and global variables of the C program, in addition to the contents of the heap (pointer dereferences) that the base level separation logic can see. The *canonical form* of a lifted assertion is $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$, where $\vec{P}$ is a list of propositions (Prop), where $\vec{Q}$ is a list of local-variable definitions (localdef), and $\vec{R}$ is a list of base-level assertions (mpred). Each list is semicolon-separated.

Lifted assertions can occur in other forms than canonical form; in fact, anything of type $\text{environ} \rightarrow \text{mpred}$ is a lifted assertion. But canonical form is most convenient for forward symbolic execution (Hoare-logic rules).

The existential quantifier $\text{EX}$ can also be used on canonical forms, e.g., $\text{EX } x:T, \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$.

Entailments in canonical form are normally of the form, $\text{ENTAIL } \Delta, PQR \vdash PQR'$, where $PQR$ is a lifted assertion in canonical form, $PQR'$ is a lifted assertion not necessarily in canonical form, and $\Delta$ is a type context. The $\vdash$ operator is written $\vdash -$ in Coq.

This notation is equivalent to $(\text{tc_environ } \Delta \&\& PQR) \vdash PQR'$. That is, $\Delta$ just provides extra assertions on the left-hand side of the entailment.
19 EX, Intros, Exists

In a canonical-form lifted assertion, existentials can occur at the outside, or in one of the base-level conjuncts within the SEP clause. This assertion has both:

\[ \text{ENTAIL } \Delta, \]
\[ \text{EX } x:Z, \]
\[ \text{PROP}(0 \leq x) \text{ LOCAL}(\text{temp } _i (\text{Vint } (\text{Int.repr } x))) \]
\[ \text{SEP}(\text{EX } y:Z, !! (x < y) \&\& \text{data.at } \pi \text{ tint } (\text{Vint } (\text{Int.repr } y)) \text{ p}) \]
\[ \vdash \text{EX } u: Z, \]
\[ \text{PROP}(0 < u) \text{ LOCAL()} \]
\[ \text{SEP}(\text{data.at } \pi \text{ tint } (\text{Vint } (\text{Int.repr } u)) \text{ p}) \]

To prove this entailment, one can first move \( x \) and \( y \) “above the line” by the tactic \textbf{Intros} \( a \ b \):

\( a: Z \)
\( b: Z \)
\( \text{H: } 0 \leq a \)
\( \text{H0: } a < b \)

\[ \text{ENTAIL } \Delta, \]
\[ \text{PROP()} \text{ LOCAL}(\text{temp } _i (\text{Vint } (\text{Int.repr } a))) \]
\[ \text{SEP}(\text{data.at } \pi \text{ tint } (\text{Vint } (\text{Int.repr } b)) \text{ p}) \]
\[ \vdash \text{EX } u: Z, \]
\[ \text{PROP}(0 < u) \text{ LOCAL()} \]
\[ \text{SEP}(\text{data.at } \pi \text{ tint } (\text{Vint } (\text{Int.repr } u)) \text{ p}) \]

One might just as well say \textbf{Intros} \( x \ y \) to use those names instead of \( a \ b \). Note that the propositions (previously hidden inside existential quantifiers) have been moved above the line by \textbf{Intros}. Also, if there had been any separating-conjunction operators \(*\) within the SEP clause, those will be “flattened” into semicolon-separated conjuncts within SEP.
Sometimes, even when there are no existentials to introduce, one wants to move PROP propositions above the line and flatten the $\ast$ operators into semicolons. One can just say $\text{Intros}$ with no arguments to do that.

If you want to Intro an existential $\text{without}$ gratuitous PROP-introduction and $\ast$-flattening, you can just use $\text{Intro}$ a, instead of $\text{Intros}$ a.

Then, instantiate $u$ by $\text{Exists}$ b.

$$a: \mathbb{Z}$$
$$b: \mathbb{Z}$$
$$\text{H: } 0 \leq a$$
$$\text{H0: } a < b$$

ENTAIL $\Delta$,

$$\text{PROP()} \text{ LOCAL}(\text{temp } i (\text{Vint (Int.repr } a)))$$
$$\text{SEP(data}_{\text{at } \pi} \text{ tint (Vint (Int.repr } b))} \text{ p})$$
$$\vdash \text{PROP}(0 < b) \text{ LOCAL()}$$
$$\text{SEP(data}_{\text{at } \pi} \text{ tint (Vint (Int.repr } b))} \text{ p})$$

This entailment proves straightforwardly by entailer!.
Coq’s standard library has the natural numbers nat and the integers Z.

C-language integer values are represented by the type Int.int (or just int for short), which are 32-bit two’s complement signed or unsigned integers with mod-$2^{32}$ arithmetic. Chapter 47 describes the operations on the int type.

For most purposes, specifications and proofs of C programs should use Z instead of int or nat. Subtraction doesn’t work well on naturals, and that screws up many other kinds of arithmetic reasoning. Only when you are doing direct natural-number induction is it natural to use nat, and so you might then convert using Z.to_nat to do that induction.

Conversions between Z and int are done as follows:

- Int.repr: $Z \rightarrow \text{int}$.
- Int.unsigned: $\text{int} \rightarrow Z$.
- Int.signed: $\text{int} \rightarrow Z$.

with the following lemmas:

- \[
\text{Int.repr}\_\text{unsigned} \quad \frac{\text{Int.repr}(\text{Int.unsigned } z) = z}{0 \leq z \leq \text{Int.max\_unsigned}}
\]

- \[
\text{Int.unsigned}\_\text{repr} \quad \frac{\text{Int.unsigned}(\text{Int.repr } z) = z}{0 \leq z \leq \text{Int.max\_unsigned}}
\]

- \[
\text{Int.repr}\_\text{signed} \quad \frac{\text{Int.repr}(\text{Int.signed } z) = z}{\text{Int.min\_signed} \leq z \leq \text{Int.max\_signed}}
\]

- \[
\text{Int.signed}\_\text{repr} \quad \frac{\text{Int.signed}(\text{Int.repr } z) = z}{\text{Int.min\_signed} \leq z \leq \text{Int.max\_signed}}
\]

Int.repr truncates to a 32-bit twos-complement representation (losing information if the input is out of range). Int.signed and Int.unsigned are different injections back to $Z$ that never lose information.
When doing proofs about integers, the recommended proof technique is to make sure your integers never overflow. That is, if the C variable \( x \) contains the value \( \text{Vint} (\text{Int.repr } x) \), then make sure \( x \) is in the appropriate range. Let’s assume that \( x \) is a signed integer, i.e. declared in C as \( \text{int } x; \) then the hypothesis is,

\[
H: \text{Int.min}_{-}\text{signed} \leq x \leq \text{Int.max}_{-}\text{signed}
\]

If you maintain this hypothesis “above the line”, then Floyd’s tactical proof automation can solve goals such as \( \text{Int.signed} (\text{Int.repr } x) = x \). Also, to solve goals such as,

\[
\ldots \\
H2: 0 \leq n \leq \text{Int.max}_{-}\text{signed} \\
\ldots
\]

\[
\text{------------------------} \\
\text{Int.min}_{-}\text{signed} \leq 0 \leq n \\
\text{------------------------}
\]

you can use the repable\_signed tactic, which is basically just omega with knowledge of the values of \( \text{Int.min}_{-}\text{signed} \), \( \text{Int.max}_{-}\text{signed} \), and \( \text{Int.max}_{-}\text{unsigned} \).

To take advantage of this, put conjuncts into the PROP part of your function precondition such as \( 0 \leq i < n; n \leq \text{Int.max}_{-}\text{signed} \). Then the start\_function tactic will move them above the line, and the other tactics mentioned above will make use of them.

To see an example in action, look at progs/verif\_sumarray.v. The array size and index (variables size and \( i \)) are kept within bounds; but the contents of the array might overflow when added up, which is why \text{add\_elem} uses \( \text{Int.add} \) instead of \( \text{Z.add} \).
21 Values: Vint, Vptr (compcert/common/Values.v)

Definition block : Type := positive.

Inductive val: Type :=
  | Vundef: val
  | Vint: int → val
  | Vlong: int64 → val
  | Vfloat: float → val
  | Vsingle: float32 → val
  | Vptr: block → int → val.

Vundef is the undefined value—found, for example, in an uninitialized local variable.

Vint(i) is an integer value, where \( i \) is a CompCert 32-bit integer. These 32-bit integers can also represent short (16-bit) and char (8-bit) values.

Vfloat(\( f \)) is a 64-bit floating-point value. Vsingle(\( f \)) is a 32-bit floating-point value.

Vptr \( b z \) is a pointer value, where \( b \) is an abstract block number and \( z \) is an offset within that block. Different malloc operations, or different extern global variables, or stack-memory-resident local variables, will have different abstract block numbers. Pointer arithmetic must be done within the same abstract block, with \((Vptr b z) + (Vint i) = Vptr b (z + i)\). Of course, the C-language + operator first multiplies \( i \) by the size of the array-element that Vptr \( b z \) points to.

Vundef is not always treated as distinct from a defined value. For example, \( p \vdash Vint 5 \vdash p \vdash Vundef \), where \( \vdash \) is the data_at operator (Chapter 26). That is, \( p \vdash Vundef \) really means \( \exists v, p \vdash v \). Vundef could mean “truly uninitialized” or it could mean “initialized but arbitrary.”
CompCert C describes C’s type system with inductive data types.

**Inductive** signedness := Signed | Unsigned.
**Inductive** intsize := I8 | I16 | I32 | IBool.
**Inductive** floatsize := F32 | F64.

**Record** attr : Type := mk_attr {
    attr_volatile: bool; attr_alignas: option N
};
**Definition** noattr := { attr_volatile := false; attr_alignas := None |}.

**Inductive** type : Type :=
    | Tvoid: type
    | Tint: intsize → signedness → attr → type
    | Tlong: signedness → attr → type
    | Tfloat: floatsize → attr → type
    | Tpointer: type → attr → type
    | TArray: type → Z → attr → type
    | TFunction: typelist → type → calling-convention → type
    | TStruct: ident → attr → type
    | TUnion: ident → attr → type

with typelist : Type :=
    | TNil: typelist
    | TCons: type → typelist → typelist.

We have abbreviations for commonly used types:

**Definition** tint = Tint I32 Signed noattr.
**Definition** tuint = Tint I32 Unsigned noattr.
**Definition** tschar = Tint I8 Signed noattr.
**Definition** tuchar = Tint I8 Unsigned noattr.
**Definition** tarray (t: type) (n: Z) = TArray t n noattr.
**Definition** tptr (t: type) := Tpointer t noattr.
23 CompSpecs

The C language has a namespace for struct- and union-identifiers, that is, *composite types*. In this example, struct foo {int value; struct foo *tail} a,b; the “global variables” namespace contains a,b, and the “struct and union” namespace contains foo.

When you use CompCert clightgen to parse myprogram.c into myprogram.v, the main definition it produces is prog, the AST of the entire C program:

Definition prog : Clight.program := { prog_types := composites; ... | }.

To interpret the meaning of a type expression, we need to look up the names of its struct identifiers in a composite environment. This environment, along with various well-formedness theorems about it, is built from prog as follows:

Require Import floyd.proofauto. (* Import Verifiable C library *)
Require Import myprogram. (* AST of my program *)

The make_compspecs tactic automatically constructs the composite specifications from the program. As a typeclass Instance, CompSpecs is supplied automatically as an implicit argument to the functions and predicates that interpret the meaning of types:

Definition sizeof {env: composite_env} (t: type) : Z := ...
Definition data_at_ {cs: compspecs} (sh: share) (t: type) (v: val) := ...

@sizeof (@cenv_cs CompSpecs) (Tint I32 Signed noattr) = 4.
sizeof (Tint I32 Signed noattr) = 4.
sizeof (Tstruct .foo noattr) = 8.
@data_at_ CompSpecs sh t v ⊢ data_at_ sh t v

When you have two separately compiled .c files, each will have its own prog and its own compspecs. See Chapter 58.
For each C-language data type, we define a representation type, the Type of Coq values that represent the contents of a C variable of that type.

**Definition**  
reptype \{cs: compspecs\} (t: type) : Type := ...

**Lemma**  
reptype_ind: \( \forall (t: \text{type}), \) reptype t =  
match t with  
| Tvoid \( \Rightarrow \) unit  
| Tint \( \Rightarrow \) val  
| Tlong \( \Rightarrow \) val  
| Tfloat \( \Rightarrow \) val  
| Tpointer \( \Rightarrow \) val  
| Tarray t0 \( \Rightarrow \) list (reptype t0)  
| Tfunction \( \Rightarrow \) unit  
| Tstruct id \( \Rightarrow \) reptype_structlist (co-members (get_co id))  
| Tunion id \( \Rightarrow \) reptype_unionlist (co-members (get_co id))  
end

reptype_structlist is the right-associative cartesian product of all the (reptypes of) the fields of the struct. For example,

struct list \{int hd; struct list *tl;\};  
struct one \{struct list *p\};  
struct three \{int a; struct list *p; double x;\};

reptype (Tstruct _list noattr) = (val*val).  
reptype (Tstruct _one noattr) = val.  
reptype (Tstruct _three noattr) = (val*(val*val)).

We use val instead of int for the retype of an integer variable, because the variable might be uninitialized, in which case its value will be Vundef.
CompCert represents uninitialized atomic (integer, pointer, float) values as $\text{Vundef} : \text{val}$.

The dependently typed function $\text{default_val}$ calculates the undefined value for any C type:

$$\text{default_val} : \forall \{\text{cs: compspecs}\} (t : \text{type}), \text{reptype} t.$$  

For any C type $t$, the default value for variables of type $t$ will have Coq type ($\text{reptype} t$).

For example:

```c
struct list {int hd; struct list *tl;};
```

$$\text{default_val tint} = \text{Vundef}$$
$$\text{default_val (tptr tint)} = \text{Vundef}$$
$$\text{default_val (tarray tint 4)} = [\text{Vundef}; \text{Vundef}; \text{Vundef}; \text{Vundef}]$$
$$\text{default_val (tarray t n)} = \text{list_repeat (Z.to_nat n) (default_val t)}$$
$$\text{default_val (Tstruct_list noattr)} = (\text{Vundef, Vundef})$$
Consider a C program with these declarations:

```
struct list {int hd; struct list *tl;} L;
int f(struct list a[5], struct list *p) { ... }
```

Assume these definitions in Coq:

```
Definition t_list := Tstruct_list noattr.
Definition t_arr := Tarray t_list 5 noattr.
```

Somewhere inside \texttt{f}, we might have the assertion,

\begin{verbatim}
PROP() LOCAL(temp _a a, temp _p p, gvar _L L)
SEP(data_at Ews t_list (Vint (Int.repr 0), nullval) L;
    data_at π t_arr (list_repeat (Z.to-nat 5) (Vint (Int.repr 1), p)) a;
    data_at π t_list (default_val t_list) p)
\end{verbatim}

This assertion says, “Local variable \_a contains address \_a, \_p contains address \_p, global variable \_L is at address \_L. There is a struct list at \_L with permission-share Ews (“extern writable share”), whose \texttt{hd} field contains 0 and whose \texttt{tl} contains a null pointer. At address \_a there is an array of 5 list structs, each with \texttt{hd=1} and \texttt{tl=\_p}, with permission \π; and at address \_p there is a single list cell that is uninitialized\textsuperscript{1}, with permission \π.”

In pencil-and-paper separation logic, we write \(q \mapsto i\) to mean \texttt{data_at Tsh tint (Vint (Int.repr i)) q}. We write \(L \mapsto (0,\texttt{NULL})\) to mean \texttt{data_at Tsh t_list (Vint (Int.repr 0), nullval) L}. We write \(p \mapsto (_{-},\_{-})\) to mean \texttt{data_at π t_list (default_val t_list) p}.

In fact, the definition \texttt{data_at_} is useful for the situation \(p \mapsto _{-}\):

```
Definition data_at_ {cs: compspecs} sh t p := data_at sh t (default_val t) p.
```

\textsuperscript{1}Uninitialized, or initialized but we don’t know or don’t care what its value is
27 retype', repinj

The retype function maps C types to the the corresponding Coq types of (possibly uninitialized) values. When we know a variable is definitely initialized, it may be more natural to use int instead of val for integer variables, and float instead of val for double variables. The retype' function maps C types to the Coq types of (definitely initialized) values.

Definition retype' \{cs: compspecs\} (t: type) : Type := ... .

Lemma retype'_ind: \forall (t: type),
retype t =
match t with
| Tvoid \Rightarrow unit
| Tint \_ \_ \_ \Rightarrow int
| Tlong \_ \_ \Rightarrow Int64.int
| Tfloat \_ \_ \Rightarrow float
| Tpointer \_ \_ \Rightarrow pointer_val
| Tarray t0 \_ \_ \Rightarrow list (retype' t0)
| Tfunction \_ \_ \_ \Rightarrow unit
| Tstruct id \_ \Rightarrow retype'_structlist (co_members (get_co id))
| Tunion id \_ \Rightarrow retype'_unionlist (co_members (get_co id))
end

The function repinj maps an initialized value to the type of possibly uninitialized values:

Definition repinj \{cs: compspecs\} (t: type) : retype' t \rightarrow retype t := ...
The program progs/nest2.c (verified in progs/verif_nest2.v) illustrates the use of reptype' and repinj.

```c
struct a {double x1; int x2;};
struct b {int y1; struct a y2;} p;

int get(void) { int i; i = p.y2.x2; return i; }
void set(int i) { p.y2.x2 = i; }
```

Our API spec for get reads as,

**Definition** get-spec :=

```
DECLARE _get
WITH v : reptype’ t_struct_b, p : val
PRE []
  PROP() LOCAL(gvar _p p)
  SEP(data_at Ews t_struct_b (repinj _ v) p)
POST [ tint ]
  PROP() LOCAL(temp ret_temp (Vint (snd (snd v))))
  SEP(data_at Ews t_struct_b (repinj _ v) p).
```

In this program, reptype’ t_struct_b = (int*(float*int)), and repinj t_struct_b (i,(x,j)) = (Vint i, (Vfloat x, Vint j)).

One could also have specified get without reptype’ at all:

**Definition** get_spec :=

```
DECLARE _get
WITH i: Z, x: float, j: int, p : val
PRE []
  PROP() LOCAL(gvar _p p)
  SEP(data_at Ews t_struct_b (Vint (Int.repr i), (Vfloat x, Vint j)) p)
POST [ tint ]
  PROP() LOCAL(temp ret_temp (Vint j))
  SEP(data_at Ews t_struct_b (Vint (Int.repr i), (Vfloat x, Vint j)) p).
```
Consider again the example in progs/nest2.c

```c
struct a {double x1; int x2;};
struct b {int y1; struct a y2;};
```

The command `i = p.y2.x2;` does a nested field load. We call `y2.x2` the field path. The precondition for this command might include the assertion,

```plaintext
LOCAL(gvar pb pb)
SEP( data_at sh t_struct_b (y1,(x1,x2)) pb)
```

The postcondition (after the load) would include the new LOCAL fact, `temp_i x2`.

The tactic `(unfold_data_at 1%nat)` changes the SEP part of the assertion as follows:

```plaintext
SEP(field_at Ews t_struct_b (DOT_y1) (Vint y1) pb;
    field_at Ews t_struct_b (DOT_y2) (Vfloat x1, Vint x2) pb)
```

and then doing `(unfold_field_at 2%nat)` unfolds the second `field_at`,

```plaintext
SEP(field_at Ews t_struct_b (DOT_y1) (Vint y1) pb;
    field_at Ews t_struct_b (DOT_y2 DOT_x1) (Vfloat x1) pb;
    field_at Ews t_struct_b (DOT_y2 DOT_x2) (Vint x2) pb)
```

The third argument of `field_at` represents the path of structure-fields that leads to a given substructure. The empty path (nil) works too; it “leads” to the entire structure. In fact, `data_at π τ v p` is just short for `field_at π τ nil v p`.

Arrays and structs may be nested together, in which case the field path may also contain array subscripts at the appropriate places, using the notation `SUB i` along with `DOT field`. 
29 Localdefs: temp, lvar, gvar

The LOCAL part of a PROP()LOCAL()SEP() assertion is a list of localdefs that bind variables to their values or addresses.

**Inductive** localdef : Type :=
  | temp: ident → val → localdef
  | lvar: ident → type → val → localdef
  | gvar: ident → val → localdef
  | sgvar: ident → val → localdef
  | localprop: Prop → localdef.

`temp i v` binds a nonaddressable local variable `i` to its value `v`.
`lvar i t v` binds an *addressable* local variable `i` (of type `t`) to its address `v`.
`gvar i v` binds a *visible global* variable `i` to its address `v`.
`sgvar i v` binds a *possibly shadowed global* variable `i` to its address `v`.

The contents of an addressable (local or global) variable is on the heap, and can be described in the SEP clause.

```plaintext
int g=2;
int f(void) { int g; int *p = &g; g=6; return g; }
```

In this program, the global variable `g` is shadowed by the local variable `g`. In an assertion inside the function body, one could write

```plaintext
PROP() LOCAL(temp -p q; lvar -g tint q; sgvar -g p}
SEP(data_at Ews tint (Vint (Int.repr 2)) p; data_at Tsh tint (Vint (Int.repr 6)) q)
```

to describe a shadowed global variable `–g` that is still there in memory but (temporarily) cannot be referred to by its name in the C program.
Normally one does not use this tactic directly, it is invoked as the first step of entailer or entailer!

Given a lifted entailment $\text{ENTAIL } \Delta, \text{PROP}(\vec{P}) \text{ LOCAL}(\vec{Q}) \text{ SEP}(\vec{R}) \vdash S$, one often wants to prove it at the base level: that is, with all of $\vec{P}$ moved above the line, with all of $\vec{Q}$ out of the way, just considering the base-level separation-logic conjuncts $\vec{R}$.

When $\Delta, \vec{P}, \vec{Q}, \vec{R}$ are concrete, the go_lower tactic does this. Concrete means that the $\vec{P}, \vec{Q}$ are nil-terminated lists (not Coq variables) that every element of $\vec{Q}$ is manifestly a localdef (not hidden in Coq abstractions), the identifiers in $\vec{Q}$ be (computable to) ground terms, and the analogous (tree) property for $\Delta$. It is not necessary that $\Delta, \vec{P}, \vec{Q}, \vec{R}$ be fully ground terms: Coq variables (and other Coq abstractions) can appear anywhere in $\vec{P}$ and $\vec{R}$ and in the value parts of $\Delta$ and $\vec{Q}$. When the entailment is not fully concrete, or when there existential quantifiers outside PROP, the tactic old_go_lower can still be useful.

go_lower moves the propositions $\vec{P}$ above the line; when a proposition is an equality on a Coq variable, substitute the variable.

For each localdef in $\vec{Q}$ (such as temp $i$ $v$), go_lower looks up $i$ in $\Delta$ to derive a type-checking fact (such as $\text{tc_val } t$ $v$), then introduces it above the line and simplifies it. For example, if $t$ is tptr tint, then the typechecking fact simplifies to $\text{is_pointer_or_null } v$.

Then it proves the localdefs in $S$, if possible. If there are still some local-environment dependencies remaining in $S$, it introduces a variable rho to stand for the run-time environment.

The remaining goal will be of the form $\vec{R} \vdash S'$, with the semicolons in $\vec{R}$ replaced by the separating conjunction $\ast$. $S'$ is the residue of $S$ after lowering to the base separation logic and deleting its (provable) localdefs.
31 saturate_local

Normally one does not use this tactic directly, it is invoked by entailor or entailer!

To prove an entailment $R_1 * R_2 * \ldots * R_n \vdash!!(P_1' \land \ldots \land P_n') & & R_1' * \ldots * R_m'$, first extract all the local (nonspatial) facts from $R_1 * R_2 * \ldots * R_n$, use them (along with other propositions above the line) to prove $P_1' \land \ldots \land P_n'$, and then work on the separation-logic (spatial) conjuncts $R_1' * \ldots * R_n \vdash R_1' * \ldots * R_m'$.

An example local fact: data_at Ews (tarray tint n) v p \vdash!! (Zlength v = n). That is, the value v in an array “fits” the length of the array.

The Hint database saturate_local contains all the local facts that can be extracted from individual spatial conjuncts:

field_at_local_facts:

- field_at π t path v p \vdash!!(field-compatible t path p
  \land value_fits (nested_field_type t path) v)

- data_at π t v p \vdash!!(field-compatible t nil p \land value_fits t v)

memory_block_local_facts:

- memory_block π n p \vdash!! isptr p

The assertion (Zlength v = n) is actually a consequence of value_fits when t is an array type. See Chapter 33.

If you create user-defined spatial terms (perhaps using EX, data_at, etc.), you can add hints to the saturate_local database as well.

The tactic saturate_local takes a proof goal of the form $R_1 * R_2 * \ldots * R_n \vdash S$ and adds saturate-local facts for each of the $R_i$, though it avoids adding duplicate hypotheses above the line.
32 field_compatible, field_address

CompCert C light comes with an “address calculus.” Consider this example:

struct a {double x1; int x2;};
struct b {int y1; struct a y2;};
struct a *pa; int *q = &(pa→y2.x2);

Suppose the value of .pa is p. Then the value of .q is p + δ; how can we reason about δ?

Given type t such as Tstruct _b noattr, and path such as (DOT _y2 DOT _x2), then (nested_field_type t path) is the type of the field accessed by that path, in this case tint; (nested_field_offset t path) is the distance (in bytes) from the base of t to the address of the field, in this case (on a 32-bit machine) 12 or 16, depending on the field-alignment conventions of the target-machine.

On the Intel x86 architecture, where doubles need not be 8-byte-aligned, we have,

data_at π t_struct_b (i,(f,j)) p ⊢
  data_at π tint i p * data_at π t_struct_a (f,j) (offset_val p 12)

but don’t write it that way! For one thing, the converse is not valid:

data_at π tint i p * data_at π t_struct_a (f,j) (offset_val p 12)
  ∀ data_at π t_struct_b (i,(f,j)) p

The reasons: we don’t know that p + 12 satisfies the alignment requirements for struct b; we don’t know whether p + 12 crosses the end-of-memory boundary. That entailment would be valid in the presence of this hypothesis: field_compatible t_struct_b nil p : Prop.

which says that an entire struct b value can fit at address p. Note that
this is a nonspatial assertion, a pure proposition, independent of the contents of memory.

In order to assist with reasoning about reassembly of data structures, saturate_local (and therefore entailer) put field_compatible assertions above the line; see Chapter 31.

Sometimes one needs to name the address of an internal field—for example, to pass just that field to a function. In that case, one could use field_offset, but it better to use field_address:

**Definition** field_address \((t: \text{type}) (path: \text{list gfield}) (p: \text{val}) : \text{val} :=\)

\[
\begin{align*}
&\text{if field_compatible_dec } t \text{ path } p \\
&\quad \text{then offset_val (Int.repr (nested_field_offset } t \text{ path)) } p \\
&\quad \text{else Vundef}
\end{align*}
\]

That is, field_address has “baked in” the fact that the offset is “compatible” with the base address (is properly aligned, has not crossed the end-of-memory boundary). And therefore:

\[
\begin{align*}
\text{data_at } &\pi \text{ tint } i \ p \\
\ast \text{ data_at } &\pi \text{ t_struct_a } (f,j) (\text{field_address t_struct_b } (\text{DOT } y2 \text{ DOT } x2) \ p) \\
\vdash &\text{ data_at } \pi \text{ t_struct_b } (i,(f,j)) \ p
\end{align*}
\]
The spatial maps-to assertion, data_at π t v p, says that there’s a value v in memory at address p, filling the data structure whose C type is t (with permission π). A corollary is value_fits t v: v is a value that actually can reside in such a C data structure.

Value_fits is a recursive, dependently typed relation that is easier described by its induction relation; here, we present a simplified version that assumes that all types t are not volatile:

value_fits t v = tc_val' t v  (when t is an integer, float, or pointer type)
value_fits (tarray t' n) v = (Zlength v = Z.max 0 n) ∧ Forall (value_fits t') v
value_fits (Tstruct i noattr) (v₁,(v₂,(...,vₙ)))) =
  value_fits (field_type f₁ v₁) ∧ ... ∧ value_fits (field_type fₙ vₙ)
  (when the fields of struct i are f₁,...,fₙ)

The predicate tc_val' says,

**Definition** tc_val' (t: type) (v: val) :=  v ≠ Vundef → tc_val t v.

**Definition** tc_val (t: type) (v: val) :=
match t with
| Tvoid ⇒ False
| Tint sz sg _ ⇒ is_int sz sg
| Tlong _ _ ⇒ is_long
| Tfloat F32 _ ⇒ is_single
| Tfloat F64 _ ⇒ is_float
| Tpointer _ _ | Tarray _ _ | Tfunction _ _ _ ⇒ is_pointer_or_null
| Tstruct _ _ | Tunion _ _ ⇒ is.ptr
end

So, an atomic value (int, float, pointer) fits either when it is Vundef or when it type-checks. We permit Vundef to “fit,” in order to accommodate partially initialized data structures in C.
Since \( \tau \) is usually concrete, \( \text{tc_val} \ \tau \ v \) immediately unfolds to something like,

\[
\begin{align*}
\text{TC0}: & \text{is\_int l32 Signed (Vint i)} \\
\text{TC1}: & \text{is\_int l8 Unsigned (Vint c)} \\
\text{TC2}: & \text{is\_int l8 Signed (Vint d)} \\
\text{TC3}: & \text{is\_pointer\_or\_null p} \\
\text{TC4}: & \text{isptr q}
\end{align*}
\]

TC0 says that \( i \) is a 32-bit signed integer; this is a tautology, so it will be automatically deleted by \text{go\_lower}.

TC1 says that \( c \) is a 32-bit signed integer whose value is in the range of unsigned 8-bit integers (unsigned char). TC2 says that \( d \) is a 32-bit signed integer whose value is in the range of signed 8-bit integers (signed char). These hypotheses simplify to,

\[
\begin{align*}
\text{TC1}: & 0 \leq \text{Int.unsigned} \ c \leq \text{Byte.max\_unsigned} \\
\text{TC2}: & \text{Byte.min\_signed} \leq \text{Int.signed} \ c \leq \text{Byte.max\_signed}
\end{align*}
\]
The cancel tactic proves associative-commutative rearrangement goals such as \((A_1 \ast A_2) \ast ((A_3 \ast A_4) \ast A_5) \vdash A_4 \ast (A_5 \ast A_1) \ast (A_3 \ast A_2)\).

If the goal has the form \((A_1 \ast A_2) \ast ((A_3 \ast A_4) \ast A_5) \vdash (A_4 \ast B_1 \ast A_1) \ast B_2\) where there is only a partial match, then cancel will remove the matching conjuncts and leave a subgoal such as \(A_2 \ast A_3 \ast A_5 \vdash B_1 \ast B_2\).

cancel solves \((A_1 \ast A_2) \ast ((A_3 \ast A_4) \ast A_5) \vdash A_4 \ast TT \ast A_1\) by absorbing \(A_2 \ast A_3 \ast A_5\) into \(TT\). If the goal has the form

\[
F := \text{?224 : list(environ \rightarrow mpred)}
\]

\[
(A_1 \ast A_2) \ast ((A_3 \ast A_4) \ast A_5) \vdash A_4 \ast (\text{fold\_right sepcon emp} \ F) \ast A_1
\]

where \(F\) is a frame that is an abbreviation for an uninstanitated logical variable of type \text{list(environ\rightarrow mpred)}, then the cancel tactic will perform frame inference: it will unfold the definition \(F\), instantiate the variable (in this case, to \(A_2 :: A_3 :: A_5 :: \text{nil}\)), and solve the goal. The frame may have been created by \text{evar}(F: \text{list(environ\rightarrow mpred)}), as part of forward symbolic execution through a function call.

**Warning:** cancel can turn a provable entailment into an unprovable entailment. Consider this:

\[
A \ast C \vdash B \ast C
\]

\[
A \ast D \ast C \vdash C \ast B \ast D
\]

This goal is provable by first rearranging to \((A \ast C) \ast D \vdash (B \ast C) \ast D\). But cancel may aggressively cancel \(C\) and \(D\), leaving \(A \vdash B\), which is not provable. You might wonder, what kind of crazy hypothesis is \(A \ast C \vdash B \ast C\); but indeed such “context-dependent” cancellations do occur in the theory of linked lists; see ?? and PLCC Chapter 19.

**Cancel does not use \(\beta\eta\) equality, as this can sometimes be very slow.** That means sometimes cancel leaves a residual subgoal \(A \vdash A'\) where \(A =_\beta A'\), sometimes the only differences are in (invisible) implicit arguments. In any case, apply \text{derives\_refl} to solve such residual goals.
The entailer and entailer! tactics simplify (or solve entirely) entailments in either the lifted or base-level separation logic. The entailer never turns a provable entailment into an unprovable one; entailer! is more aggressive and somewhat more efficient, but sometimes turns a provable entailment into an unprovable one, especially in cases related to the WARNING on page 57; see also ???. We recommend trying entailer! first, especially where list segments are not involved.

When go_lower is applicable, the entailers start by applying it (see Chapter 30).

Then: saturate_local (see Chapter 31).

NEXT: on each side of the entailment, gather the propositions to the left: \( R_1 * (!!P_1 && (!!P_2 && R_2) ) \) becomes \( !!(P_1 \land P_2) && (R_1 * R_2) \).

Move all left-hand-side propositions above the line; substitute variables. Autorewrite with entailer_rewrite, a modest hint database. If the r.h.s. or its first conjunct is a “valid_pointer” goal (or one of its variants), try to solve it.

At this point, entailer tries normalize and (if progress) back to NEXT; entailer! applies cancel to the spatial terms and prove_it_now to each propositional conjunct.

The result is that either the goal is entirely solved, or a residual entailment or proposition is left for the user to prove.
36 Normalize

The normalize tactic performs autorewrite with norm and several other transformations. Normalize can be slow: Many of these simplifications can be done more efficiently and systematically by entailer or Intros.

The norm rewrite-hint database uses several sets of rules.

Generic separation-logic simplifications.

\[
\begin{align*}
\text{P} \ast \text{emp} &= \text{P} &\text{emp} \ast \text{P} &= \text{P} &\text{P} \&\& \text{TT} &= \text{P} &\text{TT} \&\& \text{P} &= \text{P} \\
\text{P} \&\& \text{FF} &= \text{FF} &\text{FF} \&\& \text{P} &= \text{FF} &\text{P} \ast \text{FF} &= \text{FF} &\text{FF} \ast \text{P} &= \text{FF} \\
\text{P} \&\& \text{P} &= \text{P} &\text{(EX} \_ : \_ , \text{P}) &= \text{P} &\text{local} \ \text{True} &= \text{TT} \\
\end{align*}
\]

Pull EX and !! out of *-conjunctions.

\[
\begin{align*}
\text{(EX} \ x : \text{A}, \text{P}) \ast \text{Q} &= \text{EX} \ x : \text{A}, \text{P} \ast \text{Q} &\text{(EX} \ x : \text{A}, \text{P}) \&\& \text{Q} &= \text{EX} \ x : \text{A}, \text{P} \&\& \text{Q} \\
\text{P} \ast (\text{EX} \ x : \text{A}, \text{Q}) &= \text{EX} \ x : \text{A}, \text{P} \ast \text{Q} &\text{P} \&\& (\text{EX} \ x : \text{A}, \text{Q}) &= \text{EX} \ x : \text{A}, \text{P} \&\& \text{Q} \\
\text{P} \ast (!! \text{Q} \&\& \text{R}) &= !! \text{Q} \&\& (\text{P} \ast \text{R}) &(!! \text{Q} \&\& \text{P}) \ast \text{R} &= !! \text{Q} \&\& (\text{P} \ast \text{R}) \\
\end{align*}
\]

Delete auto-provable propositions.

\[
\begin{align*}
P \rightarrow (!!! \text{P} \&\& \text{Q} = \text{Q}) &\quad P \rightarrow (!!! \text{P} = \text{TT})
\end{align*}
\]

Integer arithmetic.

\[
\begin{align*}
n + 0 &= n &0 + n &= n &n \ast 1 &= n &1 \ast n &= n &\text{sizeof tuchar} &= 1 \\
\text{align} \ n \ 1 &= n & (z > 0) \rightarrow (\text{align} \ 0 \ z = 0) & (z \geq 0) \rightarrow (\text{Z.max} \ 0 \ z = z)
\end{align*}
\]
**Int32 arithmetic.**

\[
\begin{align*}
\text{Int.sub } x \ x \ &= \ \text{Int.zero} & \text{Int.sub } x \ \text{Int.zero} \ &= \ x \\
\text{Int.add } x \ (\text{Int.neg } x) \ &= \ \text{Int.zero} & \text{Int.add } x \ \text{Int.zero} \ &= \ x \\
\text{Int.add } \text{Int.zero} \ x \ &= \ x \\
x \neq y \rightarrow \text{offset_val}(\text{offset_val } v \ i) \ j \ &= \ \text{offset_val } v \ (\text{Int.add } i \ j) \\
\text{Int.add}(\text{Int.repr } i)(\text{Int.repr } j) \ &= \ \text{Int.repr}(i + j) \\
\text{Int.add}(\text{Int.add } z \ (\text{Int.repr } i))(\text{Int.repr } j) \ &= \ \text{Int.add } z \ (\text{Int.repr}(i + j)) \\
z > 0 \rightarrow (\text{align } 0 \ z \ = \ 0) & \text{ force_int}(\text{Vint } i) \ = \ i \\
(\text{min_signed} \leq z \leq \text{max_signed}) \rightarrow \text{Int.signed}(\text{Int.repr } z) \ = \ z \\
(0 \leq z \leq \text{max_unsigned}) \rightarrow \text{Int.unsigned}(\text{Int.repr } z) \ = \ z \\
(\text{Int.unsigned } i < 2^n) \rightarrow \text{Int.zero_ext } n \ i \ = \ i \\
(-2^{n-1} \leq \text{Int.signed } i < 2^{n-1}) \rightarrow \text{Int.sign_ext } n \ i \ = \ i
\end{align*}
\]

**map, fst, snd, . . .**

\[
\begin{align*}
\text{map } f \ (x :: y) \ &= \ f \ x :: \text{map } f \ y & \text{map } \text{nil} \ = \ \text{nil} & \text{fst}(x, y) \ = \ x \\
\text{snd}(x, y) \ &= \ y & (\text{isptr } v) \rightarrow \text{force_ptr } v \ = \ v & \text{isptr } (\text{force_ptr } v) \ = \ \text{isptr } v \\
&(\text{is_pointer_or_null } v) \rightarrow \text{ptr_eq } v \ v \ = \ \text{True}
\end{align*}
\]

**Unlifting.**

\[
\begin{align*}
'f \ \rho \ &= \ f \ [\text{when } f \ \text{has arity } 0] & 'f \ a_1 \ \rho \ &= \ f \ (a_1 \ \rho) \ [\text{when } f \ \text{has arity } 1] \\\n'f \ a_1 \ a_2 \ \rho \ &= \ f \ (a_1 \ \rho) \ (a_2 \ \rho) \ [\text{when } f \ \text{has arity } 2, \text{etc.}] & (P \ast Q)\rho \ &= \ P \ \rho \ast Q \ \rho \\
(P \&\& Q)\rho \ &= \ P \ \rho \&\& Q \ \rho & (!!P)\rho \ &= \ !!P & (!!(P \land Q) = !!P \&\& !!Q \\
(\text{EX } x : A, \ P x)\rho \ &= \ \text{EX } x : A, \ P x \ \rho & (\text{EX } x : B, \ P x) = \text{EX } x : B, '(P x)) \\
'(P \ast Q) \ &= \ 'P \ast 'Q & (P \&\& Q) \ &= \ 'P \&\& 'Q
\end{align*}
\]
Type checking and miscellaneous.

\[
tc\_andp \ tc\_TT \ e = e \quad tc\_andp \ e \ tc\_TT = e
\]
\[
eval\_id \ x \ (env\_set \ \rho \ x \ v) = v
\]
\[
x \neq y \rightarrow (eval\_id \ x \ (env\_set \ \rho \ y \ v) = eval\_id \ x \ v)
\]
\[
isptr \ v \rightarrow (eval\_cast\_neutral \ v = v)
\]
\[
(\exists t. \ tc\_val t v \land is\_pointer\_type t) \rightarrow (eval\_cast\_neutral \ v = v)
\]

Expression evaluation. (autorewrite with eval, but in fact these are usually handled just by simpl or unfold.)

deref\_noload(array t n) = (fun v \Rightarrow v) \quad eval\_expr(Etempvar \ i \ t) = eval\_id \ i
\]
\[
eval\_expr(Econst\_int \ i \ t) = 'Vint \ i
\]
\[
eval\_expr(Ebinop \ op \ a \ b \ t) =
\]
\[
'(eval\_binop \ op \ (typeof \ a) \ (typeof \ b)) \ (eval\_expr \ a) \ (eval\_expr \ b)
\]
\[
eval\_expr(Eunop \ op \ a \ t) = '(eval\_unop \ op \ (typeof \ a)) \ (eval\_expr \ a)
\]
\[
eval\_expr(Ecast \ e \ t) = '(eval\_cast \ (typeof \ e) \ t) \ (eval\_expr \ e)
\]
\[
eval\_lvalue(Ederef \ e \ t) = 'force\_ptr \ (eval\_expr \ e)
\]

Function return values.

get\_result(Some \ x) = get\_result1(x) \quad 
\text{retval}(get\_result1 \ i \ \rho) = eval\_id \ i \ \rho
\]
\[
\text{retval}(env\_set \ \rho \ \text{ret}\_temp \ v) = v
\]
\[
\text{retval}(make\_args(ret\_temp :: \text{nil}) \ (v :: \text{nil}) \ \rho) = v
\]
\[
\text{ret}\_type(initialized \ i \ \Delta) = \text{ret}\_type(\Delta)
\]
Postconditions. (autorewrite with ret_assert.)

\[
\text{normal_ret_assert \enspace FF \enspace ek \enspace vl} = \text{FF}
\]
\[
\text{frame_ret_assert(normal_ret_assert \enspace P) \enspace Q} = \text{normal_ret_assert} \enspace (P \ast Q)
\]
\[
\text{frame_ret_assert \enspace P \enspace emp} = P
\]
\[
\text{frame_ret_assert \enspace P \enspace Q \enspace EK\_return} \enspace vl = P \enspace EK\_return \enspace vl \ast Q
\]
\[
\text{frame_ret_assert(loop1_ret_assert P \enspace Q) \enspace R} =
\]
\[
\text{loop1_ret_assert} \enspace (P \ast R)(\text{frame_ret_assert} \enspace Q \enspace R)
\]
\[
\text{frame_ret_assert(loop2_ret_assert P \enspace Q) \enspace R} =
\]
\[
\text{loop2_ret_assert} \enspace (P \ast R)(\text{frame_ret_assert} \enspace Q \enspace R)
\]
\[
\text{overridePost \enspace P} \enspace (\text{normal_ret_assert} \enspace Q) = \text{normal_ret_assert} \enspace P
\]
\[
\text{normal_ret_assert \enspace P} \enspace ek \enspace vl = (!!(ek = \text{EK\_normal}) && (!!vl = \text{None}) && P))
\]
\[
\text{loop1_ret_assert} \enspace P \enspace Q \enspace \text{EK\_normal} \enspace \text{None} = P
\]
\[
\text{overridePost \enspace P \enspace R} \enspace \text{EK\_normal} \enspace \text{None} = P
\]
\[
\text{overridePost \enspace P \enspace R} \enspace \text{EK\_return} = R \enspace \text{EK\_return}
\]

In addition to rewriting, normalize applies the following lemmas:

\[
P \vdash \text{TT} \quad \text{FF} \vdash P \quad P \vdash P \ast \text{TT} \quad (\forall x. (P \vdash Q)) \rightarrow (\text{EX} x : A, \ P \vdash Q)
\]
\[
(P \rightarrow (\text{TT} \vdash Q)) \rightarrow (!!P \vdash Q) \quad (P \rightarrow (Q \vdash R)) \rightarrow (!!P && Q \vdash R)
\]

and does some rewriting and substitution when \( P \) is an equality in the goal, \((P \rightarrow (Q \vdash R))\).

Given the goal \( x \rightarrow P \), where \( x \) is not a \text{Prop}, normalize avoids doing an intro. This allows the user to choose an appropriate name for \( x \).
The typechecker ensures this about C-program variables: if a variable is initialized, then it contains a value of its declared type.

Function parameters (accessed by Etempvar expressions) are always initialized. Nonaddressable local variables (accessed by Etempvar expressions) and address-taken local variables (accessed by Evar) may be uninitialized or initialized. Global variables (accessed by Evar) are always initialized.

The typechecker keeps track of the initialization status of local nonaddressable variables, conservatively: if on all paths from function entry to the current point—assuming that the conditions on if-expressions and while-expressions are uninterpreted/nondeterministic—there is an assignment to variable \( x \), then \( x \) is known to be initialized.

**Addressable local variables** do not have initialization status tracked by the typechecker; instead, this is tracked in the separation logic, by data_at assertions such as \( v \mapsto \_ \) (uninitialized) or \( v \mapsto i \) (initialized).

Proofs using the forward tactic will typically generate proof obligations (for the user to solve) of the form,

\[
\text{ENTAIL} \; \Delta, \text{PROP}(\vec{P}) \; \text{LOCAL}(\vec{Q}) \; \text{SEP}(\vec{R}) \vdash \text{PROP}(\vec{P}') \; \text{LOCAL}(\vec{Q}') \; \text{SEP}(\vec{R}')
\]

\( \Delta \) keeps track of which nonaddressable local variables are initialized; says that all references to local variables contain values of the right type; and says that all addressable locals and globals point to an appropriate block of memory.

Using go_lower or entailier on an ENTAIL goal causes a tc_val assertion to be placed above the line for each initialized tempvar. As explained at page 55, this tc_val may be simplified into an is_int hypothesis, or even removed if vacuous.
The mapsto operator (and related operators) take a *permission share*, expressing whether the mapsto grants read permission, write permission, or some other fractional permission.

![Diagram of shares]

The *top* share, written $Tsh$ or $\text{Share.top}$, gives total permission: to deallocate any cells within the footprint of this mapsto, to read, to write.

$$\text{Share.split } Tsh = (\text{Share.top})$$
$$\text{Share.split } Lsh = (a, a')$$
$$\text{Share.split } Rsh = (b, b')$$
$$a' \oplus b = c$$
$$\sqcup(c, Rsh) = a' \oplus Rsh = d$$
$$\forall sh. \text{ writable_share } sh \rightarrow \text{ readable_share } sh$$
$$\text{writable_share } Ews \rightarrow \text{ readable_share b}$$
$$\text{writable_share } d \rightarrow \text{ readable_share c}$$
$$\text{writable_share } Tsh \rightarrow \neg \text{ readable_share } Lsh$$

Any share may be split into a *left half* and a *right half*. The left and right of the top share are given distinguished names $Lsh$, $Rsh$.

The right-half share of the top share (or any share containing it such as $d$) is sufficient to grant *write permission* to the data: “the right share is the write share.” A thread of execution holding only $Lsh$—or subshares of it such as $a, a'$—can neither read or write the object, but such shares are not completely useless: holding any nonempty share prevents other threads from deallocating the object.

Any subshare of $Rsh$, in fact any share that overlaps $Rsh$, grants *read* permission to the object. Overlap can be tested using the glb (greatest
Whenever \( (\text{mapsto} \ sh \ t \ v \ w) \) holds, then the share \( sh \) must include at least a read share, thus this give permission to load memory at address \( v \) to get a value \( w \) of type \( t \).

To make sure \( sh \) has enough permission to write (i.e., \( Rsh \subset sh \), we can say \( \text{writable-share} \ sh : \text{Prop} \).

Memory obtained from \text{malloc} \) comes with the top share \( Tsh \). Writable extern global variables and stack-allocated addressable locals (which of course must not be deallocated) come with the “extern writable share” \( Ews \) which is equal to \( Rsh \). Read-only globals come with a half-share of \( Rsh \).

Sequential programs usually have little need of any shares except the \( Tsh \) and \( Ews \). However, many function specifications can be parameterized over any share (example: page ??), and this sort of generalized specification makes the functions usable in more contexts.

In C it is undefined to test deallocated pointers for equality or inequalities, so the Hoare-logic rule for pointer comparison also requires some permission-share; see page 66.
39 Pointer comparisons

In C, if \( p \) and \( q \) are expressions of type pointer-to-something, testing \( p=q \) or \( p\neq q \) is defined only if: \( p \) is NULL, or points within a currently allocated object, or points at the end of a currently allocated object; and similarly for \( q \). Testing \( p<q \) (etc.) has even stricter requirements: \( p \) and \( q \) must be pointers into the same allocated object.

Verifiable C’s enforces this by creating “type-checking” conditions for the evaluation of such pointer-comparison expressions. Before reasoning about the result of evaluating expression \( p=q \), you must first prove
\[
tc_{expr} \Delta \ (Ebinop\ Oeq\ (Etempvar\_p\ (tptr\ tint))\ (Etempvar\_q\ (tptr\ tint))),
\]
where \( tc_{expr} \) is the type-checking condition for that expression. This simplifies into an entailment with the current precondition on the left, and \( denote_{tc\_comparable}\ p\ q \) on the right.

The entailer(!) has a solver for such proof goals. It relies on spatial terms on the l.h.s. of the entailment, such as \( data_{at}\ \pi\ t\ v\ p \) which guarantees that \( p \) points to something.

The file progs/verif_ptr_compare.v illustrates pointer comparisons.
40 Proof of the reverse program

Program Logics for Certified Compilers, Chapter 3 describes the notion of list segments and their application to a proof of the list-reverse function. (Chapters 2 and 3 available free here; the whole e-book available cheap here or here; or buy the hardcover.)

In this chapter we will demonstrate this proof in Verifiable C, on the C program in progs/reverse.c. Please open your CoqIDE or Proof General to progs/verif_reverse.v.

/* reverse.c */
#include <stddef.h>

struct list {int head; struct list *tail;};

struct list three[] = { {1, three+1}, {2, three+2}, {3, NULL} };

struct list *reverse (struct list *p) {
    struct list *w, *t, *v;
    w = NULL;
    v = p;
    while (v) {
        t = v→tail; v→tail = w; w = v; v = t;
    }
    return w;
}

int main (void) {
    struct list *r; int s;
    r = reverse(three); s = sumlist(r); return s;
}

As usual, in progs/verif_reverse.v we import the clightgen-produced file reverse.v and build CompSpecs and Vprog (see page 12, Chapter 23, Chapter 42).
For the struct list used in this program, struct list {int head; struct list *tail;} we can define the notion of list segment $x \sigma \rightarrow z$ with a recursive definition:

Fixpoint lseg (sh: share) (contents: list val) (x z: val) : mpred :=
match contents with
| h::hs ⇒ (x<>z) &&
   EX y:val, data_at sh (Tstruct_list noattr) (h,y) x
   * lseg sh hs y z
| nil ⇒ (ptr_eq x z) && emp
end.

But instead, we make a general theory of list segments (over any C struct type, no matter how many fields). Here, we import the LsegSpecial module of that theory, covering the “ordinary” case appropriate for the reverse.c program.

Require Import progs.list_dt. Import LsegSpecial.

Then we instantiate that theory for our particular struct list by providing the listspec operator with the names of the struct (_list) and the link field (_tail).


All other fields (in this case, just _head) are treated as “data” fields.

Now, lseg LS $\pi \sigma p q$ is a list segment starting at pointer $p$, ending at $q$, with permission-share $\pi$ and contents $\sigma$.

In general, with multiple data fields, the type of $\sigma$ is constructed via retype (see Chapter 24). In this example, with one data field, the type of $\sigma$ computes to list val.

We’ll skip over the sumlist function and its verification.
The API spec (see also Chapter 6) for reverse is,

**Definition** `reverse_spec :=
DECLARE `reverse
WITH `sh: share, `contents: list val, `p: val
PRE [ `p OF (tptr t.struct_list) ]
  PROP(writable_share `sh)
  LOCAL(temp `p `p)
  SEP(lseg LS `sh `contents `p nullval)
POST [ (tptr t.struct_list) ]
  EX `p:val,
    PROP() LOCAL(temp ret_temp `p)
    SEP(lseg LS `sh (rev `contents) `p nullval).

The precondition says (for `p the function parameter) \( p \xrightarrow{\sigma} \text{nil} \), and the postcondition says that (for `p the return value) \( p \xrightarrow{\text{rev} \sigma} \text{nil} \). This is basically the specification given in PLCC Chapter 3, page 20.

Also, the list must have write permission (writable_share `sh), because the list-reverse is an in-place destructive update.

In your IDE, enter the Lemma `body_reverse and move after the start_function tactic. As expected, the precondition for the function-body is

\[
\text{PROP() LOCAL(temp } _p `p ) \text{ SEP(lseg LS } `sh `contents `p \text{ nullval).}
\]

After forward through two assignment statements (\( w = \text{NULL; } v = `p; \)) the LOCAL part also contains temp \( _v `p \); temp \( _w (\text{Vint (Int.repr 0))}. \)

The loop invariant for the while loop is quite similar to the one given in PLCC Chapter 3 page 20:

\[
\exists \sigma_1, \sigma_2. \ \sigma = \text{rev}(\sigma_1) \cdot \sigma_2 \wedge v \xrightarrow{\sigma_2} 0 \ast w \xrightarrow{\sigma_1} 0
\]

It’s quite typical for loop invariants to existentially quantify over the values that are different iteration-to-iteration.
**Definition** reverse_inv (sh: share) (contents: list val) : environ→mpred :=
EX cts₁: list val, EX cts₂ : list val, EX w: val, EX v: val,
PROP(contents = rev cts₁ ++ cts₂)
LOCAL(temp _w w; temp _v v)
SEP(lseg LS sh cts₁ w nullval; lseg LS sh cts₂ v nullval).

We apply forward_while with this invariant, and (as usual) we have four subgoals: (1) precondition implies loop invariant, (2) loop invariant implies typechecking of loop-termination test, (3) loop body preserves invariant, and (4) after the loop.

(1) To prove the precondition implies the loop invariant, we instantiate cts₁ with nil and cts₂ with contents; we instantiate w with NULL and v with p. But this leaves the goal,

\[ \text{ENTAIL } \Delta, \text{PROP}() \text{ LOCAL}(\text{temp }_v p; \text{temp }_w \text{nullval}; \text{temp }_p p) \]
\[ \vdash \text{PROP}(\text{contents }= \text{rev } [] ++ \text{contents}) \text{ LOCAL}(\text{temp }_w \text{nullval}; \text{temp }_v p) \]
\[ \text{SEP}(\text{lseg LS } sh \text{ contents } p \text{nullval}) \]
\[ \text{lseg LS } sh \text{ [] nullval nullval; } \text{lseg LS } sh \text{ contents } p \text{nullval}) \]

The PROP and LOCAL parts are trivially solvable by the entailer. We can remove the SEP conjunct (lseg LS sh [] nullval nullval) by rewriting in the theory of list segments:

**Lemma** lseg_eq: \( \forall (LS : \text{lists} \_\text{list} \_\text{tail}) (\pi : \text{share}) (l : \text{list} \_)(v : \text{val}), \)
\( \text{is\_pointer\_or\_null } v \rightarrow \)
\( \text{lseg LS } \pi l v v = !!(l = []) \) \&\& emp.

(2) The type-checking condition is not trivial, as it is a pointer comparison (see Chapter 39), but the entailer! solves it anyway.

(3) The loop body starts by assuming the loop invariant and the truth of the loop test. Their propositional parts have already been moved above the line at the comment (* loop body preserves invariant *). That is, HRE: isptr v says that the loop test is true, and \( \text{H}: \text{contents }= \text{rev } cts₁ ++ cts₂ \) is from the invariant.
The first statement in the loop body, \( t = v \rightarrow \text{tail} \); loads from the list cell at \( v \). But our SEP assertion for \( v \) is, \( \text{lseg} \ LS \ sh \ cts_2 \ v \ nullval \). A list-segment isn't necessarily loadable, i.e., we cannot necessarily fetch \( v \rightarrow \text{tail} \); what we need to unfold the lseg, using this lemma:

**Lemma** \( \text{lseg} \text{-nonnull} \): \( \forall (LS: \text{listspec} \_\text{list} \_\text{tail}) \ (\pi: \text{share}) \ (l: \text{list} _) \ v, \)
\[ \text{typed-true} \ (\text{tptr} \ t\_\text{struct} \_\text{list}) \ v \rightarrow \]
\[ \text{lseg} \ LS \ \pi \ l \ v \ nullval = \]
\[ \text{EX} \ h: -, \text{EX} \ r: -, \text{EX} \ y: \text{val}, \]
\[ \text{!!}(l = h :: r \land \text{is-pointer_or_null} \ y) \land \]
\[ \text{list} \_\text{cell} \ LS \ \pi \ h \ x \star \]
\[ \text{field} \_\text{at} \ \pi \ t\_\text{struct} \_\text{list} \ (\text{SUB} \ _\text{tail}) \ y \ x \star \]
\[ \text{lseg} \ LS \ \pi \ r \ y \ z. \]

That is, if \( v \neq \text{nullval} \), then the list-segment \( v \rightarrow \text{nullval} \) is not empty: there exists a record \( x \rightarrow (h, y) \) and a residual list \( y \rightarrow \text{nullval} \). Actually, here it is more convenient to use a corollary of this lemma, \( \text{semax} \_\text{lseg} \_\text{nonnull} \), that is adapted to unfolding the first lseg in the SEP clause of a semax precondition. The typed-true premise solves easily by entailer!.

**Now that the first list-cell is unfolded**, it's easy to go forward through the four commands of the loop body. Now we are (* at end of loop body, re-establish invariant *).

We choose appropriate values to instantiate the existentials: \( \text{Exists} \ (h::cts_1,r,v,y) \).

Note that for some reason the four separate \( \text{EX} \) quantifiers have been uncurried into a single 4-tuple \( \text{EX} \); this may be adjusted in a future version of Verifiable C. Then entailer! leaves two subgoals:

\[ \text{----------------------------------------------(1/2)} \]
\[ \text{rev} \ cts_1 ++ h :: r = (\text{rev} \ cts_1 ++ [h]) ++ r \]
\[ \text{----------------------------------------------(2/2)} \]
\[ \text{list} \_\text{cell} \ LS \ sh \ h \ v \star \text{field} \_\text{at} \ sh \ t\_\text{struct} \_\text{list} \ (\text{DOT} \ _\text{tail}) \ w \ v \]
\[ \star \text{lseg} \ LS \ sh \ cts_1 \ w \ nullval \]
\[ \vdash \text{lseg} \ LS \ sh \ (h :: cts_1) \ v \ nullval \]
Indeed, entail! always leaves at most two subgoals: at most one propositional goal, and at most one cancellation (spatial) goal. Here, the propositional goal is easily dispatched in the theory of (Coq) lists.

The second subgoal requires unrolling the r.h.s. list segment, which we do with lseg_unroll. Then we appropriately instantiate some existentials, call on the entail! again, and the goal is solved.

(4) After the loop, we must prove that the loop invariant and not the loop-test condition is a sufficient precondition for the next statement(s). In this case, the next statement is a return; one can always go forward through a return, but now we have to prove that our current assertion implies the function postcondition. This is fairly straightforward.
In progs/verif_reverse.v, in the Lemma body_sumlist, move to the comment (* Prove that loop body preserves invariant *), and then three or four lines to just before assert_PROP.

This proof state is very similar to the one in the loop body of the body_reverse lemma (page 71):

```
contents, cts1, cts2 : list int; p, t, y : val; i : int
SH : readable_share sh
HRE : isptr t
H : contents = cts1 ++ i :: cts2
H1 : is_pointer_or_null y
```

```
semax Delta
  (PROP () LOCAL(temp _t t; temp _s (Vint (sum_int cts1))))
  SEP(list_cell LS sh (Vint i) t; field_at sh list_struct [StructField_tail] y t;
       lseg LS sh (map Vint cts2) y nullval; lseg LS sh (map Vint cts1) p t))
  h = t → head; ...
```

**POSTCONDITION**

Here, the operator list_cell (from the general theory of list segments) describes “all the fields but the link.” In our particular LS there is exactly one data field, which fact we state as a lemma:

**Lemma** list_cell_eq: ∀ sh i p ,

sepalg.nonidentity sh →
field_compatible t_struct_list [] p →
list_cell LS sh (Vint i) p =
field_at sh t_struct_list (DOT_head) (Vint i) p.

To rewrite by list_cell_eq, we need to get a field_compatible fact above the line. Such facts are promiscuously introduced by saturate_local as part of calling entail!, but we are not currently proving an entailment. No matter; we can prove one artificially:
assertPROP (field_compatible t_struct_list nil t) as FC by entailer!.

The assert_prop tactic creates an ENTAIL proof goal with the current semax precondition on the left, and the named proposition on the right. That proposition is then put above the line; really this is a use of the rule of consequence. It’s an easy way to get this field_compatible fact above the line.
42 Global variables

In the C language, “extern” global variables live in the same namespace as local variables, but they are shadowed by any same-name local definition. In the C light operational semantics, global variables live in the same namespace as addressable local variables (both referenced by the expression-abstract-syntax constructor Evar), but in a different namespace from nonaddressable locals (expression-abstract-syntax constructor Etempvar).\(^1\)

In the program-AST produced by clightgen, globals (and their initializers) are listed as Gvars in the progdefs. These are accessed (automatically) in two ways by the Verifiable C program logic. First, their names and types are gathered into Vprog as shown on page 12 (try the Coq command Print Vprog to see this list). Second, their initializers are translated into data_at conjuncts of separation logic as part of the main_pre definition (see page 32).

When proving semax_body for the main function, the start_function tactic takes these definitions from main_pre and puts them in the precondition of the function body. In VST version 1.6, in some cases this is done using the more-primitive mapsto operator\(^2\), in other cases it uses the higher-level (and more standard) data_at\(^3\).

---

\(^1\)This difference in namespace treatment cannot matter in a program translated by CompCert clightgen from C, because no as-translated expression will exercise the difference.

\(^2\)For example, examine the proof state in progs/verif_reverse.v immediately after start_function in Lemma body_main; and see the conversion to data_at done by the setup_globals lemma in that file.

\(^3\)For example, examine the proof state in progs/verif_sumarray.v immediately after start_function in Lemma body_main.
43 For loops

The C-language for loop has the general form,

\[
\text{for (init; test; incr) body}
\]

To solve a proof goal of this form (or when this is followed by other statements in sequence), use the tactic

\[
\text{forward_for Inv PreIncr PostCond}
\]

where Inv, PreIncr, PostCond are assertions (in PROP/LOCAL/SEP form):

Inv is the loop invariant, that holds immediately after the init command is executed and before each time the test is done; PreIncr is the invariant that holds immediately after the loop body and right before the incr;

PostCond is the assertion that holds after the loop is complete (whether by a break statement, or the test evaluating to false).

In VST 1.6, forward_for is slightly broken, but you can use forward_for_simple_bound (described below) or forward_while (since a for loop is often a special case of a while loop).

Many for-loops have this special form,

\[
\text{for (init; id < hi; id++) body}
\]

such that the expression hi will evaluate to the same value every time around the loop. This upper-bound expression need not be a literal constant, it just needs to be invariant. Then you can use the tactic,

\[
\text{forward_for_simple_bound n (EX i:Z, PROP(\vec{P}) \ LOCAL(\vec{Q}) \ SEP(\vec{R}))}
\]

where n is the upper bound: a Coq value of type Z such that hi will evaluate to n. The loop invariant is given by the expression...
(EX i:Z, PROP(P) LOCAL(Q) SEP(R), where \( i \) is the value (in each iteration) of the loop iteration variable \( id \). This tactic generates simpler subgoals than the general forward_for tactic.

When the loop has the form,  
\[
\text{for (id=lo; id < hi; id++) body}
\]
where \( lo \) is a literal constant, then the forward_for_simple_bound tactic will generate slightly simpler subgoals.
**44 Manipulating preconditions**

In some cases you cannot go forward until the precondition has a certain form. For example, to go forward through $t=\mathit{v}\rightarrow\mathit{tail}$; there must be a data_at or field_at in the SEP clause of the precondition that gives a value for $\_\mathit{tail}$ field of $t$. page 71 describes a situation where a list segment had to be unfolded to expose such a SEP conjunct.

Faced with the proof goal, $\text{semax } \Delta \left(\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})\right) \ c \ \text{Post}$ where $\text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(\vec{R})$ does not match the requirements for forward symbolic execution, you have several choices:

- Use the rule of consequence explicitly:
  apply $\text{semax\_pre}$ with $\text{PROP}(\vec{P}')\text{LOCAL}(\vec{Q}')\text{SEP}(\vec{R}')$, then prove $\text{ENTAIL } \Delta, \vec{P};\vec{Q};\vec{R} \vdash \vec{P}';\vec{Q}';\vec{R}'$.
- Use the rule of consequence implicitly, by using tactics (page 79) that modify the precondition.
- Do rewriting in the precondition, either directly by the standard rewrite and change tactics, or by normalize (page 59).
- Extract propositions and existentials from the precondition, by using $\text{Intros}$ (page 37) or normalize.
- Flatten stars into semicolons, in the SEP clause, by $\text{Intros}$.
- Use the freezer (page 100) to temporarily “frame away” spatial conjuncts.
TACTICS FOR MANIPULATING PRECONDITIONS. In many of these tactics we select specific conjuncts from the SEP items, that is, the semicolon-separated list of separating conjuncts. These tactics refer to the list by zero-based position number, 0, 1, 2, ... .

For example, suppose the goal is a semax or entailment containing \( \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(a; b; c; d; e; f; g; h; i; j) \). Then:

- **focus\_SEP \( i \), \( j \), \( k \).** Bring items \#\( i \), \( j \), \( k \) to the front of the SEP list.
  - \( \text{focus\_SEP} \ 5 \). \ \( \text{results in} \ \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(f; a; b; c; d; e; g; h; i; j) \).
  - \( \text{focus\_SEP} \ 0 \). \ \( \text{results in} \ \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(a; b; c; d; e; f; g; h; i; j) \).
  - \( \text{focus\_SEP} \ 1, 3 \). \ \( \text{results in} \ \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(b; d; a; c; e; f; g; h; i; j) \).
  - \( \text{focus\_SEP} \ 3, 1 \). \ \( \text{results in} \ \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(d; b; a; c; e; f; g; h; i; j) \).

- **gather\_SEP \( i \), \( j \), \( k \).** Bring items \#\( i \), \( j \), \( k \) to the front of the SEP list and conjoin them into a single element.
  - \( \text{gather\_SEP} \ 5 \). \ \( \text{results in} \ \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(f; a; b; c; d; e; g; h; i; j) \).
  - \( \text{gather\_SEP} \ 1, 3 \). \ \( \text{results in} \ \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(b* d; a; c; e; f; g; h; i; j) \).
  - \( \text{gather\_SEP} \ 3, 1 \). \ \( \text{results in} \ \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(d* b; a; c; e; f; g; h; i; j) \).

- **replace\_SEP \( i \), \( R \).** Replace the \( i \)th element the SEP list with the assertion \( R \), and leave a subgoal to prove.
  - \( \text{replace\_SEP} \ 3, R \). \ \( \text{results in} \ \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(a; b; c; R; e; f; g; h; i; j) \).
  - With subgoal \( \text{PROP}(\vec{P})\text{LOCAL}(\vec{Q})\text{SEP}(d) \vdash R \).

- **replace\_in\_pre \( S \), \( S' \).** Replace \( S \) with \( S' \) anywhere it occurs in the precondition then leave \( (\vec{P}; \vec{Q}; \vec{R}) \vdash (\vec{P}; \vec{Q}; \vec{R})[S'/S] \) as a subgoal.

- **frame\_SEP \( i \), \( j \), \( k \).** Apply the frame rule, keeping only elements \( i \), \( j \), \( k \) of the SEP list. See Chapter 45.
45 The Frame rule

Separation Logic supports the Frame rule,

\[
\text{Frame} \quad \frac{\{P\} \{Q\}}{\{P \ast F\} \{Q \ast F\}}
\]

To use this in a forward proof, suppose you have the proof goal,

\[
\text{semax} \; \Delta \; \text{PROP}(\vec{P}) \text{LOCAL}(\vec{Q}) \text{SEP}(R_0; R_1; R_2) \; c_1; c_2; c_3 \; \text{Post}
\]

and suppose you want to “frame out” \( R_2 \) for the duration of \( c_1; c_2 \), and have it back again for \( c_3 \). First you rewrite by \text{seq_assoc} to yield the goal

\[
\text{semax} \; \Delta \; \text{PROP}(\vec{P}) \text{LOCAL}(\vec{Q}) \text{SEP}(R_0; R_1; R_2) \; (c_1; c_2); c_3 \; \text{Post}
\]

Then \text{eapply} \text{semax_seq'} to peel off the first command \((c_1; c_2)\) in the new sequence:

\[
\text{semax} \; \Delta \; \text{PROP}(\vec{P}) \text{LOCAL}(\vec{Q}) \text{SEP}(R_0; R_1; R_2) \; c_1; c_2 \; ?88
\]

\[
\text{semax} \; \Delta' \; ?88 \; c_3 \; \text{Post}
\]

Then \text{frame_SEP 0 2} to retain only \( R_0; R_2 \).

\[
\text{semax} \; \Delta \; \text{PROP}(\vec{P}) \text{LOCAL}(\vec{Q}) \text{SEP}(R_0; R_2) \; c_1; c_2 \; \ldots
\]

Now you'll see that (in the precondition of the second subgoal) the unification variable \(?88\) has been instantiated in such a way that \( R_2 \) is added back in.
46 malloc/free

If your program uses malloc or free, you must declare and specify these as external functions. But even then, free is difficult to specify: “How do it know?” the size of the object being freed?

The answer is that the malloc/free system maintains an implicit extra field (before the official “beginning” of the object) with the length. One could indeed reason about this in separation logic, but for some applications it is overkill.

For simpler-to-specify memory allocation, you may want to change the interface of the free function. We do this in our example definitions of malloc and free in progs/queue.c and their specifications in progs/verif_queue.v.
The VST program logic uses CompCert’s 32-bit integer type.

**Inductive** comparison := Ceq | Cne | Clt | Cle | Cgt | Cge.

Int.wordsizer: nat = 32.
Int.modulus : Z = 2\(^{32}\).
Int.max_unsigned : Z = 2\(^{32} - 1\).
Int.max_signed : Z = 2\(^{31} - 1\).
Int.min_signed : Z = \(-2^{31}\).

Int.int : Type.
Int.unsigned : int \to Z.
Int.signed : int \to Z.
Int.repr : Z \to int.

Int.zero := Int.repr 0.

(* Operators of type int-\to\int-\to bool *)
Int.eq  Int.lt  Int.ltu  Int.cmp(c:comparison)  Int.cmpu(c:comparison)

(* Operators of type int-\to\int *)
Int.neg  Int.not

(* Operators of type int-\to\int-\to\int *)
Int.add  Int.sub  Int.mul  Int.divs  Int.mods  Int.divu  Int.modu
Int.and  Int.or  Int.xor  Int.shl  Int.shru  Int.shr  Int.rol  Int.ror  Int.rolm

**Lemma** eq-dec: \(\forall (x \, y: \int), \{x = y\} + \{x <> y\}\).
**Theorem** unsigned_range: \(\forall i, 0 \leq \text{unsigned} \, i < \text{modulus}\).
**Theorem** unsigned_range_2: \(\forall i, 0 \leq \text{unsigned} \, i \leq \text{max unsigned}\).
**Theorem** signed_range: \(\forall i, \text{min signed} \leq \text{signed} \, i \leq \text{max signed}\).
**Theorem** repr_unsigned: \(\forall i, \text{repr} \, (\text{unsigned} \, i) = i\).
**Lemma** repr_signed: \(\forall i, \text{repr} \, (\text{signed} \, i) = i\).
**Theorem** unsigned_repr:
\(\forall z, 0 \leq z \leq \text{max unsigned} \to \text{unsigned} \, (\text{repr} \, z) = z\).
Theorem signed_repr:
\[ \forall z, \text{min}\_\text{signed} \leq z \leq \text{max}\_\text{signed} \rightarrow \text{signed}\ (\text{repr}\ z) = z. \]

Theorem signed_eq_unsigned:
\[ \forall x, \text{unsigned}\ x \leq \text{max}\_\text{signed} \rightarrow \text{signed}\ x = \text{unsigned}\ x. \]

Theorem unsigned_zero: unsigned zero = 0.

Theorem unsigned_one: unsigned one = 1.

Theorem signed_zero: signed zero = 0.

Theorem eq_sym: \[ \forall x, y, \text{eq}\ x\ y = \text{eq}\ y\ x. \]

Theorem eq_spec: \[ \forall (x, y : \text{int}), \text{if eq}\ x\ y\ \text{then} \ x = y \ \text{else} \ x \not\ = y. \]

Theorem eq_true: \[ \forall x, \text{eq}\ x\ x = \text{true}. \]

Theorem eq_false: \[ \forall x, y, x \not\ = y \rightarrow \text{eq}\ x\ y = \text{false}. \]

Theorem add_unsigned: \[ \forall x, y, \text{add}\ x\ y = \text{repr}\ (\text{unsigned}\ x + \text{unsigned}\ y). \]

Theorem add_signed: \[ \forall x, y, \text{add}\ x\ y = \text{repr}\ (\text{signed}\ x + \text{signed}\ y). \]

Theorem add_commut: \[ \forall x, y, \text{add}\ x\ y = \text{add}\ y\ x. \]

Theorem add_zero: \[ \forall x, \text{add}\ x\ zero = x. \]

Theorem add_zero_l: \[ \forall x, \text{add}\ zero\ x = x. \]

Theorem add_assoc: \[ \forall x, y, z, \text{add}\ (\text{add}\ x\ y)\ z = \text{add}\ x\ (\text{add}\ y\ z). \]

Theorem neg_repr: \[ \forall z, \text{neg}\ (\text{repr}\ z) = \text{repr}\ (-z). \]

Theorem neg_zero: neg zero = zero.

Theorem neg_involutive: \[ \forall x, \text{neg}\ (\text{neg}\ x) = x. \]

Theorem neg_add_distr: \[ \forall x, y, \text{neg}\ (\text{add}\ x\ y) = \text{add}\ (\text{neg}\ x)\ (\text{neg}\ y). \]

Theorem sub_zero_l: \[ \forall x, \text{sub}\ x\ zero = x. \]

Theorem sub_zero_r: \[ \forall x, \text{sub}\ zero\ x = \text{neg}\ x. \]

Theorem sub_add_opp: \[ \forall x, y, \text{sub}\ x\ y = \text{add}\ x\ (\text{neg}\ y). \]

Theorem sub_idem: \[ \forall x, \text{sub}\ x\ x = \text{zero}. \]

Theorem sub_add_l: \[ \forall x, y, z, \text{sub}\ (\text{add}\ x\ y)\ z = \text{add}\ (\text{sub}\ x\ z)\ y. \]

Theorem sub_add_r: \[ \forall x, y, z, \text{sub}\ x\ (\text{add}\ y\ z) = \text{add}\ (\text{sub}\ x\ z)\ (\text{neg}\ y). \]

Theorem sub_shifted: \[ \forall x, y, z, \text{sub}\ (\text{add}\ x\ z)\ (\text{add}\ y\ z) = \text{sub}\ x\ y. \]

Theorem sub_signed: \[ \forall x, y, \text{sub}\ x\ y = \text{repr}\ (\text{signed}\ x\ \text{-signed}\ y). \]
**Theorem** mul\_commut: $\forall x, y$, $\text{mul } x \text{ y } = \text{mul } y \text{ x}$.

**Theorem** mul\_zero: $\forall x$, $\text{mul } x \text{ zero } = \text{zero}$.

**Theorem** mul\_one: $\forall x$, $\text{mul } x \text{ one } = x$.

**Theorem** mul\_assoc: $\forall x, y, z$, $\text{mul } (\text{mul } x \text{ y)} z = \text{mul } x (\text{mul } y z)$.

**Theorem** mul\_add\_distr\_l: $\forall x, y, z$, $\text{mul } (\text{add } x \text{ y)} z = \text{add } (\text{mul } x z) (\text{mul } y z)$.

**Theorem** mul\_signed: $\forall x, y$, $\text{mul } x \text{ y } = \text{repr } (\text{signed } x * \text{ signed } y)$.

and many more axioms for the bitwise operators, shift operators, signed/unsigned division and mod operators.
48 CompCert C abstract syntax

The CompCert verified C compiler translates standard C source programs into an abstract syntax for CompCert C, and then translates that into abstract syntax for C light. Then VST Separation Logic is applied to the C light abstract syntax. C light programs proved correct using the VST separation logic can then be compiled (by CompCert) to assembly language.

C light syntax is defined by these Coq files from CompCert:

**Integers.** 32-bit (and 8-bit, 16-bit, 64-bit) signed/unsigned integers.

**Floats.** IEEE floating point numbers.

**Values.** The val type: integer + float + pointer + undefined.

**AST.** Generic support for abstract syntax.

**Ctypes.** C-language types and structure-field-offset computations.

**Clight.** C-light expressions, statements, and functions.

You will see C light abstract syntax constructors in the Hoare triples (semax) that you are verifying. We summarize the constructors here.

**Inductive** expr : Type :=

(* 1 *) | Econst_int: int → type → expr
(* 1.0 *) | Econst_float: float → type → expr (* double precision *)
(* 1.0f0 *) | Econst_single: float → type → expr (* single precision *)
(* 1L *) | Econst_long: int64 → type → expr
(* x *) | Evar: ident → type → expr
(* x *) | Etempvar: ident → type → expr
(* *e *) | Ederef: expr → type → expr
(* &e *) | Eaddrof: expr → type → expr
(* ~e *) | Eunop: unary_operation → expr → type → expr
(* e+e *) | Ebinop: binary_operation → expr → expr → type → expr
(* (int)e *) | Ecast: expr → type → expr
(* e.f *) | Efield: expr → ident → type → expr.
**Inductive** unary-operation := Onotbool | Onotint | Oneg | Oabsfloat.

**Inductive** binary-operation := Oadd | Osub | Omul | Odiv | Omod
| Oand | Oor | Oxor | Oshl | Oeq | One | Olt | Ogt | Ole | Oge.

**Inductive** statement : Type :=

(* / ** / ; * *) | Sskip : statement

(* E₁=E₂; *) | Sassign : expr → expr → statement (* memory store *)

(* x=E; *) | Sset : ident → expr → statement (* tempvar assign *)

(* x=f(...); *) | Scall: option ident → expr → list expr → statement

(* x=b(...); *) | Sbuiltin: option ident → external-function → typelist →
list expr → statement

(* s₁; s₂ *) | Ssequence : statement → statement → statement

(* if() else {} *) | Sifthenelse : expr → statement → statement → statement

(* for (;;s₂) s₁ *) | Sloop: statement → statement → statement

(* break; *) | Sbreak : statement

(* continue; *) | Scontinue : statement

(* return E; *) | Sreturn : option expr → statement

| Sswitch : expr → labeled_statements → statement

| Slabel : label → statement → statement

| Sgoto : label → statement.
49 C light semantics

The operational semantics of C light statements and expressions is given in compcert/cfrontend/Clight.v. We do not expose these semantics directly to the user of Verifiable C. Instead, the statement semantics is reformulated as semax, an axiomatic (Hoare-logic style) semantics. The expression semantics is reformulated in veric/expr.v and veric/Cop2.v as a computational big-step evaluation semantics. In each case, a soundness proof relates the Verifiable C semantics to the CompCert Clight semantics.

Rules for semax are given in veric/SeparationLogic.v—but the user rarely uses these rules directly. Instead, derived lemmas regarding semax are proved in floyd/* .v and Floyd’s forward tactic applies them (semi)automatically.

The following functions (from veric/expr.v) define expression evaluation:

```plaintext
eval_id {CS: compspecs} (id: ident) : environ → val.
    (* evaluate a tempvar *)
eval_var {CS: compspecs} (id: ident) (ty: type) : environ → val.
    (* evaluate an lvar or gvar, addressable local or global variable *)
eval_cast (t t’: type) (v: val) : val.
    (* cast value v from type t to type t’, but beware! There are three types involved, including native type of v. *)
eval_unop (op: unary-operation) (t1 : type) (v1 : val) : val.
eval_binop {CS: compspecs} (op:binary_operation) (t1 t2: type) (v1 v2: val): val.
eval_lvalue {CS: compspecs} (e: expr) : environ → val.
    (* evaluate an l-expression, one that denotes a loadable/storable place*)
eval_expr {CS: compspecs} (e: expr) : environ → val.
    (* evaluate an r-expression, one that is not storable *)
```

The environ argument is for looking up the values of local and global variables. However, in most cases where Verifiable C users see eval_lvalue or eval_expr—in subgoals generated by the forward tactic—all the variables have already been substituted by values. Thus the environment is not
The expression-evaluation functions call upon several helper functions from veric/Cop2.v:

\[\begin{align*}
\text{sem\_cast}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_cast\_\star} &\quad (* \text{several helper functions for sem\_cast} *) \\
\text{bool\_val}: \text{type} \rightarrow \text{val} \rightarrow \text{option bool}. \\
\text{bool\_val\_\star} &\quad (* \text{helper functions} *) \\
\text{sem\_notbool}: \text{type} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_neg}: \text{type} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_sub} \{\text{CS: compspecs}\}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_sub\_\star} &\quad (* \text{helper functions} *) \\
\text{sem\_add} \{\text{CS: compspecs}\}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_add\_\star} &\quad (* \text{helper functions} *) \\
\text{sem\_mul}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_div}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_mod}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_and}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_xor}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_shl}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_shr}: \text{type} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_cmp}: \text{comparison} \rightarrow \text{type} \rightarrow \text{type} \rightarrow (\ldots) \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_unary\_operation}: \text{unary\_operation} \rightarrow \text{type} \rightarrow \text{val} \rightarrow \text{option val}. \\
\text{sem\_binary\_operation} \{\text{CS: compspecs}\}: \\
\quad \text{binary\_operation} \rightarrow \text{type} \rightarrow \text{type} \rightarrow \text{mem} \rightarrow \text{val} \rightarrow \text{val} \rightarrow \text{option val}.
\end{align*}\]

The details are not so important to remember. The main point is that Coq expressions of the form sem\_... should simplify away, provided that their arguments are instantiated with concrete operators, concrete constructors \text{Vint/Vptr/Vfloat}, and concrete C types. The int values (etc.) carried inside \text{Vint/Vptr/Vfloat} do not need to be concrete: they can be Coq variables. This is the essence of proof by symbolic execution.
Many of the Hoare rules, such as the one on page ??,

\[ \text{semax}_\text{set}_\text{forward} \quad \Delta \vdash \{\triangleright P\} \quad x := e \quad \exists v. x = (e[v/x]) \land P[v/x] \]

have the operator \( \triangleright \) (pronounced “later”) in their precondition.

The modal assertion \( \triangleright P \) is a slightly weaker version of the assertion \( P \). It is used for reasoning by induction over how many steps left we intend to run the program. The most important thing to know about \( \triangleright \) later is that \( P \) is stronger than \( \triangleright P \), that is, \( P \vdash \triangleright P \); and that operators such as \( * \), \&\&, \text{ALL} (and so on) commute with later: \( \triangleright (P * Q) = (\triangleright P) * (\triangleright Q) \).

This means that if we are trying to apply a rule such as \text{semax}_\text{set}_\text{forward}; and if we have a precondition such as

\[ \text{local} (\text{tc}\_\text{expr} \Delta e) \&\& \triangleright \text{local} (\text{tc}\_\text{temp}\_\text{id} \text{id} t \Delta e) \&\& (P_1 * \triangleright P_2) \]

then we can use the rule of consequence to weaken this precondition to

\[ \triangleright (\text{local} (\text{tc}\_\text{expr} \Delta e) \&\& \text{local} (\text{tc}\_\text{temp}\_\text{id} \text{id} t \Delta e) \&\& (P_1 * P_2)) \]

and then apply \text{semax}_\text{set}_\text{forward}. We do the same for many other kinds of command rules.

This weakening of the precondition is done automatically by the forward tactic, as long as there is only one \( \triangleright \) later in a row at any point among the various conjuncts of the precondition.

A more sophisticated understanding of \( \triangleright \) is needed to build proof rules for recursive data types and for some kinds of object-oriented programming; see PLCC Chapter 19.
51 Nested Loads

This experimental appeared in VST release 1.5, but is broken in VST 1.6.

To handle assignment statements with nested loads, such as \(x[i]=y[i]+z[i]\); the recommended method is to break it down into smaller statements compatible with separation logic: \(t=y[i]; u=z[i]; x[i]=t+u;\). However, sometimes you may be proving correctness of preexisting or machine-generated C programs. Verifiable C has an \textbf{experimental} nested-load mechanism to support this.

We use an expression-evaluation relation \(e \Downarrow v\) which comes in two flavors:

\[
\begin{align*}
\text{rel_expr} &: \text{expr} \rightarrow \text{val} \rightarrow \rho \rightarrow \text{mpred}. \\
\text{rel_lvalue} &: \text{expr} \rightarrow \text{val} \rightarrow \rho \rightarrow \text{mpred}.
\end{align*}
\]

The assertion \(\text{rel_expr} e v \rho\) says, “expression \(e\) evaluates to value \(v\) in environment \(\rho\) and in the current memory.” The \text{rel_lvalue} evaluates the expression as an \(l\)-value, to a pointer to the data.

Evaluation rules for \text{rel_expr} are listed here:

\[
\begin{align*}
\text{rel_expr.const.int} &: \forall (i : \text{int}) \quad \tau \quad (P : \text{mpred}) \quad (\rho : \text{environ}), \quad P \vdash \text{rel_expr} (\text{Econst.int} i \quad \tau) \quad (\text{Vint} i) \quad \rho. \\
\text{rel_expr.const.float} &: \forall (f : \text{float}) \quad \tau \quad P \quad (\rho : \text{environ}), \quad P \vdash \text{rel_expr} (\text{Econst.float} f \quad \tau) \quad (\text{Vfloat} f) \quad \rho. \\
\text{rel_expr.const.long} &: \forall (i : \text{int64}) \quad \tau \quad P \quad \rho, \quad P \vdash \text{rel_expr} (\text{Econst.long} i \quad \tau) \quad (\text{Vlong} i) \quad \rho. \\
\text{rel_expr.tempvar} &: \forall (id : \text{ident}) \quad \tau \quad (v : \text{val}) \quad P \quad \rho, \quad \text{Map.get (te_of } \rho) \quad \text{id} = \text{Some } v \rightarrow \quad P \vdash \text{rel_expr} (\text{Etempvar} id \quad \tau) \quad v \quad \rho. \\
\text{rel_expr.addrof} &: \forall (e : \text{expr}) \quad \tau \quad (v : \text{val}) \quad P \quad \rho, \quad P \vdash \text{rel_lvalue} e \quad v \quad \rho \rightarrow \quad P \vdash \text{rel_expr} (\text{Eaddrof} e \quad \tau) \quad v \quad \rho. \\
\text{rel_expr.unop} &: \forall P \quad (e_1 : \text{expr}) \quad (v_1 : \text{val}) \quad \tau \quad \text{op} \quad \rho, \quad P \vdash \text{rel_expr} e_1 \quad v_1 \quad \rho \rightarrow
\end{align*}
\]
Cop.sem_unary_operation $op$ $v_1$ (typeof $e_1$) = Some $v$ $→$
P $⊢$ rel_expr (Eunop $e_1$ $τ$) $v$ $ρ$.

rel_expr.binop: $∀$($e_1$ $e_2$ : expr) ($v_1$ $v_2$ $v$ : val) $τ$ $op$ $P$ $ρ$,
P $⊢$ rel_expr $e_1$ $v_1$ $ρ$ $→$
P $⊢$ rel_expr $e_2$ $v_2$ $ρ$ $→$
($∀$ $m$ : Memory.Mem.mem,
Cop.sem_binary_operation $op$ $v_1$ $e$ (typeof $e_1$) $v_2$ (typeof $e_2$) $m$ = Some $v$) $→$
P $⊢$ rel_expr (Ebinop $e_1$ $e_2$ $τ$) $v$ $ρ$.

rel_expr.cast: $∀$($e_1$ : expr) ($v_1$ $v$ : val) $τ$ $P$ $ρ$,
P $⊢$ rel_expr $e_1$ $v_1$ $ρ$ $→$
Cop.sem_cast $v_1$ (typeof $e_1$) $τ$ = Some $v$ $→$
P $⊢$ rel_expr (Ecast $e_1$ $τ$) $v$ $ρ$.

rel_expr.lvalue: $∀$($a$ : expr) ($sh$ : Share.t) ($v_1$ $v_2$ : val) $P$ $ρ$,
P $⊢$ rel_lvalue $a$ $v_1$ $ρ$ $→$
P $⊢$ mapsto $sh$ (typeof $a$) $v_1$ $v_2$ * $TT$ $→$
$v_2$ $<>$ Vundef $→$
P $⊢$ rel_expr $a$ $v_2$ $ρ$.

rel_lvalue.local: $∀$($id$ : ident) $τ$ ($b$ : block) $P$ $ρ$,
P $⊢$!!(Map.get (ve_of $ρ$) id = Some ($b$, $τ$)) $→$
P $⊢$ rel_lvalue (Evar $id$ $τ$) ($Vptr$ $b$ Int.zero) $ρ$.

rel_lvalue.global: $∀$($id$ : ident) $τ$ ($v$ : val) $P$ $ρ$,
P
$⊢$!!(Map.get (ve_of $ρ$) id = None $∧$
Map.get (ge_of $ρ$) id = Some ($v$, $τ$)) $→$
P $⊢$ rel_lvalue (Evar $id$ $τ$) ($v$ $ρ$).

rel_lvalue.deref: $∀$($a$ : expr) ($b$ : block) ($z$ : int) $τ$ $P$ $ρ$,
P $⊢$ rel_expr $a$ ($Vptr$ $b$ $z$) $ρ$ $→$
P $⊢$ rel_lvalue (Ederef $a$ $τ$) ($Vptr$ $b$ $z$) $ρ$.

rel_lvalue.field_struct: $∀$($i$ $id$ : ident) $τ$ ($e$ : block) ($z$ : int) ($fList$ : fieldlist) att ($δ$),
typeof $e$ = Tstruct id $fList$ att $→$
field_offset $i$ $fList$ = Errors.OK $δ$ $→$
P $⊢$ rel_expr $e$ ($Vptr$ $b$ $z$) $ρ$ $→$
P $⊢$ rel_lvalue (Efield $e$ $i$ $τ$) ($Vptr$ $b$ (Int.add $z$ (Int.repr $δ$))) $ρ$. 
The primitive nested-load assignment rule is,

**Axiom** semax_loadstore:
\[
\forall v_0 \, v_1 \, v_2 \, \Delta \, e_1 \, e_2 \, sh \, P \, P',
\]
\[
\text{writable_share sh} \rightarrow \\
P \vdash \text{!! (tc_val (typeof e_1) v_2)} \\
& \& \text{rel_lvalue e_1 v_1} \\
& \& \text{rel_expr (Ecast e_2 (typeof e_1)) v_2} \\
& \& \text{`(mapsto sh (typeof e_1) v_1 v_0) \ast P')} \rightarrow \\
\text{semax } \Delta \text{ (▷ P) (Sassign e_1 e_2)} \\
(\text{normal_ret_assert (`(mapsto sh (typeof e_1) v_1 v_2) \ast P')}).
\]

*but do not use this rule!* It is best to use a derived rule, such as,

**Lemma** semax_loadstore_array:
\[
\forall n \, v_i \, lo \, hi \, t_1 \, (\text{contents: } Z \rightarrow \text{retype } t_1) \, v_1 \, v_2 \, \Delta \, e_1 \, e_i \, e_2 \, sh \, P \, Q \, R, \\
\text{retype } t_1 = \text{val} \rightarrow \\
\text{type_is_by_value } t_1 \rightarrow \\
\text{legal_alignas_type } t_1 = \text{true} \rightarrow \\
\text{typeof } e_1 = \text{tptr } t_1 \rightarrow \\
\text{typeof } e_i = \text{tint} \rightarrow \\
\text{PROP}_x P \, (\text{LOCAL}_x Q \, (\text{SEP}_x R)) \\
\vdash \text{rel_expr e_1 v_1} \\
& \& \text{rel_expr e_i (Vint (Int.repr vi))} \\
& \& \text{rel_expr (Ecast e_2 t_1) v_2} \rightarrow \\
\text{nth_error } R \, n = \text{Some (`(array_at t_1 sh contents lo hi v_1))} \rightarrow \\
\text{writable_share sh} \rightarrow \\
\text{tc_val t_1 v_2} \rightarrow \\
\text{in_range lo hi vi} \rightarrow \\
\text{semax } \Delta \text{ (▷ PROP}_x P \, (\text{LOCAL}_x Q \, (\text{SEP}_x R)))} \\
(\text{Sassign (Ederef (Ebinop Oadd e_1 e_i (tptr t_1)) t_1) e_2}) \\
(\text{normal_ret_assert} \\
(\text{PROP}_x P \, (\text{LOCAL}_x Q \, (\text{SEP}_x} \\
(\text{replace_nth n R} \\
`\text{(array_at t_1 sh (upd contents vi (valinject v_2)) lo hi v_1))}))))).
Proof-automation support is available for `semax_loadstore_array` and `rel_expr`, in the form of the `forward_nl` (for “forward nested loads”) tactic. For example, with this proof goal,

```plaintext
semax Delta
( PROP ()
  LOCAL(`(eq (Vint (Int.repr i))) (eval_id .i); `(eq x) (eval_id .x);
  `(eq y) (eval_id .y); `(eq z) (eval_id .z))
  SEP(`(array_at tdouble Tsh (Vfloat oo fx) 0 n x);
  `(array_at tdouble Tsh (Vfloat oo fy) 0 n y);
  `(array_at tdouble Tsh (Vfloat oo fz) 0 n z))
) (Ssequence
  (Sassign (* x[i] = y[i] + z[i]; *)
    (Ederef (Ebinop Oadd (Etempvar .x (tptr tdouble)) (Etempvar .i tint)
      (tptr tdouble)) tdouble)
    (Ebinop Oadd
      (Ederef (Ebinop Oadd (Etempvar .y (tptr tdouble)) (Etempvar .i tint)
        (tptr tdouble)) tdouble)
      (Ederef (Ebinop Oadd (Etempvar .z (tptr tdouble)) (Etempvar .i tint)
        (tptr tdouble)) tdouble))
  )
) (MORE_COMMANDS)
(POSTCONDITION)
```

the tactic-application `forward_nl` yields the new proof goal,

```plaintext
semax Delta
( PROP ()
  LOCAL(`(eq (Vint (Int.repr i))) (eval_id .i); `(eq x) (eval_id .x);
  `(eq y) (eval_id .y); `(eq z) (eval_id .z))
  SEP
  (`(array_at tdouble Tsh
    ( upd (Vfloat oo fx) i (Vfloat (Float.add (fy i) (fz i)))) 0 n x);
  `(array_at tdouble Tsh (Vfloat oo fy) 0 n y);
  `(array_at tdouble Tsh (Vfloat oo fz) 0 n z))
) (MORE_COMMANDS)
(POSTCONDITION)
```
This chapter is needed only by “power users.”

Assertions in our Hoare triple of separation are presented as \( \text{env} \rightarrow \text{mpred} \), that is, functions from environment to memory-predicate, using our natural deduction system \( \text{NatDed} \) and separation logic \( \text{SepLog} \).

Given a separation logic over a type \( B \) of formulas, and an arbitrary type \( A \), we can define a lifted separation logic over functions \( A \rightarrow B \). The operations are simply lifted pointwise over the elements of \( A \). Let \( P, Q : A \rightarrow B \), let \( R : T \rightarrow A \rightarrow B \) then define,

\[
\begin{align*}
(P \& \& Q) : A \rightarrow B & := \text{fun } a \Rightarrow Pa \& \& Qa \\
(P \parallel Q) : A \rightarrow B & := \text{fun } a \Rightarrow Pa \parallel Qa \\
(\exists x. R(x)) : A \rightarrow B & := \text{fun } a \Rightarrow \exists x. Rxa \\
(\forall x. R(x)) : A \rightarrow B & := \text{fun } a \Rightarrow \forall x. Rxa \\
(P \rightarrow Q) : A \rightarrow B & := \text{fun } a \Rightarrow Pa \rightarrow Qa \\
(P \vdash Q) : A \rightarrow B & := \forall a. Pa \vdash Qa \\
(P \ast Q) : A \rightarrow B & := \text{fun } a \Rightarrow Pa \ast Qa \\
(P \negast Q) : A \rightarrow B & := \text{fun } a \Rightarrow Pa \negast Qa
\end{align*}
\]

In Coq we formalize the typeclass instances LiftNatDed, LiftSepLog, etc., as shown below. For a type \( B \), whenever \( \text{NatDed} \) \( B \) and \( \text{SepLog} \) \( B \) (and so on) have been defined, the lifted instances \( \text{NatDed} (A \rightarrow B) \) and \( \text{SepLog} (A \rightarrow B) \) (and so on) are automagically provided by the typeclass system.

**Instance** LiftNatDed(A B: Type){ND: NatDed B}: NatDed (A→B):=
\[
\begin{align*}
(*\text{andp}*) (\text{fun } P \ Q x \Rightarrow \text{andp} (P \ x) (Q \ x)) \\
(*\text{orp}*) (\text{fun } P \ Q x \Rightarrow \text{orp} (P \ x) (Q \ x)) \\
(*\text{exp}*) (\text{fun } \{T\} (F: T \rightarrow A \rightarrow B) (a: A) \Rightarrow \text{exp} (\text{fun } x \Rightarrow F \ x \ a)) \\
(*\text{allp}*) (\text{fun } \{T\} (F: T \rightarrow A \rightarrow B) (a: A) \Rightarrow \text{allp} (\text{fun } x \Rightarrow F \ x \ a)) \\
(*\text{imp}*) (\text{fun } P \ Q x \Rightarrow \text{imp} (P \ x) (Q \ x)) \\
(*\text{prop}*) (\text{fun } P x \Rightarrow \text{prop} P) \\
(*\text{derives}*) (\text{fun } P Q \Rightarrow \forall x, \text{derives} (P \ x) (Q \ x))
\end{align*}
\]
**Instance** LiftSepLog (A B: Type) {NB: NatDed B}{SB: SepLog B}

: SepLog (A → B).

apply (mkSepLog (A → B) - (fun ρ ⇒ emp)

(fun P Q ρ ⇒ P ρ * Q ρ) (fun P Q ρ ⇒ P ρ -* Q ρ)).

(* fill in proofs here *)

In particular, if $P$ and $Q$ are functions of type $\text{environ} \rightarrow \text{mpred}$ then we can write $P * Q$, $P & & Q$, and so on.

Consider this assertion:

```markdown
fun ρ ⇒ mapsto sh tint (eval_id _x ρ) (eval_id _y ρ)
     * mapsto sh tint (eval_id _u ρ) (Vint Int.zero)
```

which might appear as the precondition of a Hoare triple. It represents $(x \mapsto y) * (u \mapsto 0)$ written in informal separation logic, where $x, y, u$ are C-language variables of integer type. Because it can be inconvenient to manipulate explicit lambda expressions and explicit environment variables $\rho$, we may write it in lifted form,

```
`(mapsto sh tint) (eval_id _x) (eval_id _y)
* `(mapsto sh tint) (eval_id _u) `(Vint Int.zero)
```

Each of the first two backquotes lifts a function from type $\text{val} \rightarrow \text{val} \rightarrow \text{mpred}$ to type $(\text{environ} \rightarrow \text{val}) \rightarrow (\text{environ} \rightarrow \text{val}) \rightarrow (\text{environ} \rightarrow \text{mpred})$, and the third one lifts from $\text{val}$ to $\text{environ} \rightarrow \text{val}$. 
Aside from the standard operators and axioms of separation logic, the core separation logic has just two primitive spatial (memory) predicates:

**Parameter** address\_mapsto:

- memory\_chunk $\to$ val $\to$ share $\to$ share $\to$ address $\to$ mpred.

**Parameter** func\_ptr : funspec $\to$ val $\to$ mpred.

\( \text{func\_ptr } \phi \ v \) means that value \( v \) is a pointer to a function with specification \( \phi \); see ???.

address\_mapsto expresses what is typically written \( x \mapsto y \) in separation logic, that is, a singleton heap containing just value \( y \) at address \( x \).

From this, we construct two low-level derived forms:

\[
\text{mapsto } (sh:share) (t:type) (v w: val) : \text{mpred} \quad \text{describes a singleton heap with just one value } w \text{ of (C-language) type } t \text{ at address } v, \text{ with permission-share } sh.
\]

\[
\text{mapsto\_} (sh:share) (t:type) (v:val) : \text{mpred} \quad \text{describes an } \text{uninitialized} \text{ singleton heap with space to hold a value of type } t \text{ at address } v, \text{ with permission-share } sh.
\]

From these primitives, field\_at and data\_at are constructed.
54 Function pointers

Parameter func_ptr : funspec → val → mpred.
Definition func_ptr’ f v := func_ptr f v && emp.

func_ptr φ v means that value v is a pointer to a function with specification φ.
func_ptr’ φ v is a form more suitable to be a conjunct of a SEP clause.

Verifiable C’s program logic is powerful enough to reason expressively about function pointers (see PLCC Chapters 24 and 29). However, the Floyd proof-automation system does not have much support for proving such programs at present.
These axioms of separation logic are often useful, although generally it is the automation tactics (entailer, cancel) that apply them.

\[
pred\_ext: \text{P} \vdash \text{Q} \rightarrow \text{Q} \vdash \text{P} \\ derives\_refl: \text{P} \vdash \text{P}.
\]

\[
derives\_trans: \text{P} \vdash \text{Q} \rightarrow \text{Q} \vdash \text{R} \\ andp\_right: \text{X} \vdash \text{P} \\ andp\_left1: \text{P} \vdash \text{R} \\ andp\_left2: \text{Q} \vdash \text{R} \\ orp\_left: \text{P} \vdash \text{R} \\ orp\_right1: \text{P} \vdash \text{Q} \\ orp\_right2: \text{P} \vdash \text{Q} \\ exp\_right: \forall \{B: \text{Type}\}(x:B)(\text{P}:\text{mpred})(\text{Q}: B \rightarrow \text{mpred}), \\ \text{P} \vdash \text{Q} x \\ exp\_left: \forall \{B: \text{Type}\}(\text{P}: B \rightarrow \text{mpred})(\text{Q}:\text{mpred}), \\ (\forall x, \text{P} x \vdash \text{Q}) \\ allp\_left: \forall \{B\}(\text{P}: B \rightarrow \text{mpred}) \times Q, \text{P} x \vdash Q \\ allp\_right: \forall \{B\}(\text{P}: \text{mpred})(\text{Q}:B \rightarrow \text{mpred}), \\ (\forall v, \text{P} v \vdash \text{Q} v) \\ prop\_left: \forall (\text{P}: \text{Prop}) Q, (\text{P} \rightarrow (\text{TT} \vdash \text{Q})) \\ prop\_right: \forall (\text{P}: \text{Prop}) Q, P \rightarrow (Q \vdash \text{!!P}). \\ not\_prop\_right: \forall (\text{P}:\text{mpred})(\text{Q}:\text{Prop}), (Q \rightarrow (\text{P} \vdash \text{FF})) \\ sepcon\_assoc: (\text{P} \times Q) \times R = P \times (Q \times R). \\ sepcon\_comm: P \times Q, P \times Q = Q \times P. \\ sepcon\_andp\_prop: P \times (\text{!!Q} \&\& R) = \text{!!Q} \&\& (P \times R). \\ derives\_extract\_prop: (P \rightarrow Q \vdash \text{R}) \\ derives\_derives: P \vdash P' \\ Q \vdash Q' \\ P \times Q \vdash P' \times Q'.
56 Obscure higher-order axioms

\text{imp_andp_adjoint}: P \& \& Q \vdash R \iff P \vdash (Q \rightarrow R).

\text{wand_sepcon_adjoint}: P \ast Q \vdash R \iff P \vdash Q \ast R.

\text{ewand_sepcon}: (P \ast Q) \rightarrow R = P \rightarrow (Q \rightarrow R).

\text{ewand_TT_sepcon}: \forall (P, Q, R: A),
\begin{align*}
(P \ast Q) \& \& (R \rightarrow TT) & \vdash (P \& \&(R \rightarrow TT)) \ast (Q \& \& (R \rightarrow TT)).
\end{align*}

\text{exclude_elsewhere}: P \ast Q \vdash (P \& \&(Q \rightarrow TT)) \ast Q.

\text{ewand_conflict}: P \ast Q \vdash FF \rightarrow P \& \&(Q \rightarrow R) \vdash FF

\text{now_later}: P \vdash \triangledown P.

\text{later_K}: \triangledown (P \rightarrow Q) \vdash (\triangledown P \rightarrow \triangledown Q).

\text{later_allp}: \forall T (F: T \rightarrow \text{mpred}), \triangledown (\text{ALL } x:T, F x) = \text{ALL } x:T, \triangledown (F x).

\text{later_exp}: \forall T (F: T \rightarrow \text{mpred}), \text{EX } x:T, \triangledown (F x) \vdash \triangledown (\text{EX } x: F x).

\text{later_exp'}: \forall T (\text{any}:T) F, \triangledown (\text{EX } x: F x) = \text{EX } x:T, \triangledown (F x).

\text{later_imp}: \triangledown (P \rightarrow Q) = (\triangledown P \rightarrow \triangledown Q).

\text{loeb}: \triangledown P \vdash P \rightarrow TT \vdash P.

\text{later_sepcon}: \triangledown (P \ast Q) = \triangledown P \ast \triangledown Q.

\text{later_wand}: \triangledown (P \rightarrow Q) = \triangledown P \rightarrow \triangledown Q.

\text{later_ewand}: \triangledown (P \rightarrow Q) = (\triangledown P) \rightarrow (\triangledown Q).
57 Proving larg(ish) programs

When your program is not all in one .c file, see also Chapter 58. Whether or not your program is all in one .c file, you can prove the individual function bodies in separate .v files. This uses less memory, and (on a multicore computer with parallel make) saves time. To do this, put your API spec (up to the construction of Gprog in one file; then each semax_body proof in a separate file that imports the API spec.

EXTRACTION OF SUBORDINATE SEMAX-GOALS. To ease memory pressure and recompilation time, it is often advisable to partition the proof of a function into several lemmas. Any proof state whose goal is a semax-term can be extracted as a stand-alone statement by invoking tactic semax_subcommand V G F. The three arguments are as in the statement of surrounding semax-body lemma, i.e. are of type varspecs, funspecs, and function.

The subordinate tactic mkConciseDelta V G F Δ can also be invoked individually, to concisely display the type context Δ as the application of a sequence of initializations to the host function’s func_tycontext.

THE FREEZER. A distinguishing feature of separation logic is the frame rule, i.e. the ability to modularly verify a statement w.r.t. its minimal resource footprint. Unfortunately, being phrased in terms of the syntactic program structure, the standard frame rule does not easily interact with forward symbolic execution as implemented by the Floyd tactics (and many other systems), as these continuously rearrange the associativity of statement sequencing to peel off the redex of the next forward, and (purposely) hide the program continuation as the abbreviation MORE_COMMANDS.

Resolving this conflict, Floyd’s freezer abstraction provides a means for flexible framing, by implementing a veil that opaquely hides selected items of a SEP clause from non-symbolic treatment by non-freezer tactics.
The freezer abstraction consists of two main tactics, \textit{freeze} \( N \, F \) and \textit{thaw} \( F \), where \( N : \text{list nat} \) and \( F \) is a user-supplied (fresh) Coq name. The result of applying \textit{freeze} \([i_1; \ldots; i_n] \, F\) to a semax goal is to remove items \( i_1, \ldots, i_n \) from the precondition’s SEP clause, inserting the item \( \text{FRZL} \, F \) at the head of the SEP list, and adding a hypothesis \( F := \text{abbreviate} \) to Coq’s proof context.

The term \( \text{FRZL} \, F \) participates symbolically in all non-freezer tactics just like any other SEP item, so can in particular be canceled, and included in a function call’s frame. Unfolding a freezer is not tied to the associativity structure of program statements but can be achieved by invoking \textit{thaw} \( F \), which simply replaces \( \text{FRZL} \, F \) by the the list of \( F \)’s constiuents. As multiple freezers can coexists and freezers can be arbitrarily nested, SEP-clauses \( R \) effectively contain forests of freezers, each constituent being thawable independently and freezer-level by freezer-level.

Wrapping single \textit{forward} or \textit{forward\_call} commands in a freezer often speeds up the processing time noticably, as invocations of subordinate tactics \textit{entailer}, \textit{cancel}, etc. are supplied with smaller and more symbolic proof goals. In our experience, applying the freezer throughout the proof of an entire function body typically yields a speedup of about 30% on average with improvements of up to 55% in some cases, while also easing the memory pressure and freeing up valuable real estate on the user’s screen.

A more invasive implementation of a freezer-like abstraction would refine the \( \text{PROP}(P) \, \text{LOCAL}(Q) \, \text{SEP}(R) \) structure to terms of the form \( \text{PROP}(P) \, \text{LOCAL}(Q) \, \text{SEP}(R) \, \text{FR}(H) \) where \( H : \text{list mpred} \). Again, terms in \( H \) would be treated opaquely by all tactics, and freezing/thawing would correspond to transfer rules between \( R \) and \( H \). In either case, forward symbolic execution is reconciled with the frame rule, and the use of the mechanism is sound engineering practice as documentation of programmer’s insight is combined with performance improvements.
58 Separate compilation, semax_ext

What to do when your program is spread over multiple .c files.

**CODE PREPARATION.** In order to separate the namespaces of multiple files compiled by CompCert’s clightgen tool, it is necessary to apply

```
python fix_clightgen.py file1.v ...fileN.v
```

The script reads in the named files, concisely renames variables etc by making up new positives, and writes the modified files back to the given names.
59 Catalog of tactics / lemmas

Below is an alphabetic catalog of the major floyd tactics. In addition to short descriptions, the entries indicate whether a tactic (or tactic notation) is typically user-applied [u], primarily of internal use [i] or is expected to be used at development-time but unlikely to appear in a finished proof script [d]. We also mention major interdependencies between tactics, and their points of definition.

cancel (tactic; page 57) Deletes identical spatial conjuncts from both sides of a base-level entailment.

derives_refl (lemma) \( A \vdash A \). Useful after cancel to handle \( \beta\eta \)-equality; see page 57.

derives_refl' (lemma) \( A = B \rightarrow A \vdash B \).

entailer (tactic; page 58, page 24) Proves (lifted or base-level) entailments, possibly leaving a residue for the user to prove. The more aggressive entailer! should usually be used, but it sometimes turns a provable goal into an unprovable goal.

drop_LOCAL \( n \) (tactic, where \( n : \text{nat} \)). Removes the \( n \)th entry of a the LOCAL block of a semax or ENTAIL precondition.

forward (tactic; page ??) Do forward Hoare-logic proof through one C statement (assignment, break, continue, return).

forward_call ARGS (tactic; page ??, page 33) Forward Hoare-logic proof through one C function-call, where \( \text{ARGS} \) is a witness for the WITH clause of the funspec.

forward_for (tactic, page 76) This tactic does not work well in VST 1.6. Use forward_for_simple_bound when applicable, or else forward_while.

forward_for_simple_bound \( n \ Inv \) (tactic, page 76) When a for-loop has the form for \( \text{(init; } i < hi; i++ \text{)} \) where the upper-bound \( hi \) is a loop-invariant expression, then use this tactic: \( n \) is the value of \( hi \), and \( \text{Inv} \) is the loop invariant, which must start with an \text{EX} that binds the iteration-dependent value of variable \( i \).

forward_seq (tactic)

mkConciseDelta \( V \ G \ F \ \Delta \) (tactic) Applicable to a proof state with a semax goal. Simplifies the \( \Delta \) component to the application of a
sequence of initializations to the host function’s func_tycontext. Used to prepare the current proof goal for abstracting/factoring out as a separate lemma.

**semax_subcommand** $V G F$ (tactic) Applicable to a proof state with a semax goal. Extracts the current proof state as a stand-alone statement that can be copy-and pasted to a separate file. The three arguments should be copied from the statement of surrounding semax-body lemma: $V: \text{vars} \text{specs}, G: \text{fun} \text{spec} \text{s}, F: \text{function}$. 